COP 6405: Analysis of Algorithms FINAL REVIEW - PART 1; Fall 2019

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1. **Short Questions**
   (a) Prove that DFS in undirected graphs labels does not lead to cross edges or forward edges.
   (b) Assuming that DFS in undirected graphs gives DFS numbers to all vertices in the graph, back edges go from higher numbered vertices to lower numbered vertices. True or False? Why?
   (c) For each stage of Prim's and Kruskal's algorithms shown in FIgures 23.4 and 23.5, show the cuts where the edges picked were the minimum weight edge.
2. Modify the pseudocode for depth-first search so that it prints out every edge in the directed graph G, together with its type. Show what modifications, if any, you need to make if G is undirected.
3. A *directed acyclic graph* is a directed graph with no directed cycles. A topological sort of a directed acyclic graph is a linear ordering of all its vertices such that if the graph contains an edge (u, v) then u appears v before in the ordering. (If the graph contains a cycle, then no linear ordering is possible.)

   One way to perform topological sorting on a directed acyclic graph is to repeatedly find a vertex of in-degree 0, output it, and remove it and all of its outgoing edges from the graph. Explain how to implement this idea so that it runs in time $O(m+n)$, where m is the number of edges and n is the number of vertices.
4. Prove/argue that the biconnected components of an undirected graph form a tree if you build a graph where the vertices are the components and the edges link biconnected components that share a vertex.
5. Solve the **Euler Tour** problem 22-3 on page 623.
6. Read problem 22-4 on page 623. We have solved it in class. What problem does it remind you of?
7. Solve 23.2-4 and 23.2-5 on page 637.
8. Solve 23.2-7 on page 637.
9. Show that adding an additional edge to any spanning tree of an undirected connected graph forms exactly one cycle.
10. Show that adding a non-tree edge e to a minimum spanning tree T results in exactly one cycle and that $w(e)$ is at least as large as the weight of any other edge on that cycle.