Robustness Analysis of Source Localization Using Gaussianity Measure

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***Abstract*-Nowadays, the source localization has been widely applied for wireless sensor networks. The Gaussian mixture model has been adopted for maximum-likelihood (ML) source localization schemes. However, this model does not match the statistics of the real data in practice. In this paper, we study the probability density function of the sensor signals and demonstrate that the distribution is not Gaussian. We propose to employ the Gaussianity test based on the bootstrap algorithm to quantify the departure of Gaussianity for the received signals added with different kinds of noise. Our proposed Gaussianity test can be used as the robustness figure for evaluating the prevalent ML source localization schemes.**

Keywords-robustness analysis, source localization, Gaussianity.

I. INTRODUCTION

Recently, the sensor networks have been the ubiquitous wireless technology. The sensor signal processing techniques play a crucial role therein. Among these sensor signal processing methods, the *near-field wideband source localization* has drawn a lot of research interest in the signal processing and wireless communications. Extensive studies can be found in [1-6] for the wide-band source localization. Among them, the maximum-likelihood (ML) approach has been regarded as the optimal and robust scheme for the coherent source signals [1]. Then *Expectation-Maximization* (EM)-based localization algorithm is proposed to mitigate the computational complexity [7].

All the existing localization methods are based on the Gaussianity assumption, which could only be justified asymptotically (the number of signals gets large). This assumption is not realistic especially when the signal sample size is finite and the signal-to-noise ratio is high [8]. Hence, in the real situation, the mismatch of the underlying statistical model would degrade the ML source localization performance and steer the performance away from the optimality. However, no existing literature has ever been dedicated to the studies of this crucial statistical mismatch problem in source localization. Therefore, in this paper, we would like to address this mismatch issue. Two questions related to source localization will be discussed and answered here: (*i*) how do we measure the statistical mismatch? (*ii*) how does the statistical mismatch affect the localization performance?

For the first question, we propose to employ the higher-order statistics to resolve in this paper. Here we manifest the conceptual establishment towards the answer to this question. By means of the Edgeworth expansion, we may evaluate the mismatch between the Gaussian assumption and the true statistics directly extracted from the data [9, 10]. This approach belongs to the *non-Gaussianity test* (NGT). On the other hand, the Gaussianity test (GT) originated from the simple moment test proposed by Pearson [11]. Later on, Moulines categorized this GT approach into the time-domain and the frequency-domain Gaussianity tests [12]: the time-domain GT can be achieved via parameter estimation, characteristic function test, etc. [13,14]; the frequency-domain test can be based on the bispectral information, which arose from Hinich's test [15]. Recently, the measures of Kullback Leibler divergence (KLD) and Shannon entropy were also widely used for non-Gaussianity tests [16, 17]. The aforementioned GT and NGT techniques pave a substantial foundation for the analyses of the new applications in wireless communications and signal processing. Therefore, we propose to employ the GT approach to derive the important *robustness analysis of the ML source localization* in this paper (NGT is just the duality of GT). To the best of our knowledge, this is the first ever attempt to establish such a robustness analysis for the source localization problems. Based on our proposed GT studies, we can relate the source localization performance to the statistical mismatch measure and it addresses the second essential question.

The rest of this paper is organized as follows. The ML source localization problem and our previous EM algorithm for combating this problem are presented in Section II. In Section III, we define a new measure of the statistical mismatch between the underlying probability model and the actual data for source localization, based on the bispectrum. To illustrate the effectiveness of our proposed mismatch measure, we investigate the measure values and the localization performances over the simulations involving different kinds of noise statistics. The results are shown in Section IV. Conclusion will be drawn in Section V.

Notations: $\underline{\alpha}$ denotes a column vector and $\overline{\beta}$ denotes a matrix. R is the set of all real numbers and C is the set of all complex numbers. The symbol $\equiv$ represents the mathematical definition. The statistical expectation operation is denoted as $E[\ ]$.

II. ML SOURCE LOCALIZATION

According to [1], we consider a randomly distributed array of P sensors to collect the data from M sources. Since the sources are assumed to be in the near field, the signal

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gains are different across the sensors. Then, the signal collected by the p^{th} sensor at time n is given by

$$x_p(n) = \sum_{m=1}^{M} a_p^{(m)} s_0^{(m)}\left(n - t_p^{(m)}\right) + w_p(n), \quad (1)$$

for $n = 0,1,\cdots,L-1$, $p = 1,\cdots,P$, $m = 1,\cdots,M$, where $a_p^{(m)}$ is the signal gain level of the m^{th} source at the p^{th} sensor; $s_0^{(m)}$ denotes the m^{th} source signal waveform; $t_p^{(m)}$ is the propagation delay in samples between the p^{th} sensor and the m^{th} source; $w_p(n)$ represents the independently identically distributed zero-mean noise process. Moreover, several location parameters can be specified as follows:

$t_p^{(m)} = \dfrac{\left\| \underline{r_s}^{(m)} - \underline{r_p} \right\|}{v}$: propagation delay,

$\underline{r_s}^{(m)} \in \mathrm{R}^{2\times 1}$: the m^{th} source location,

$\underline{r_p}$: the p^{th} sensor location,

v : the propagation speed in meters/sec. Taking the discrete Fourier transform (DFT) of both sides of Eq. (1), we have

$$\underline{X}(k) = \overline{D}(k)\,\underline{S_0}(k) + \underline{U}(k),\ k = 0,1,\cdots,N-1, \quad (2)$$

where

$\underline{X}(k) \equiv \left[X_1(k) \ \cdots \ X_P(k)\right]^T \in \mathrm{C}^{P\times 1}$ ($X_p(k)$ is the k^{th} DFT point of $x_p(n)$, $p = 1,\cdots,P$); $\overline{D}(k) \equiv \left[\underline{d}^{(1)}(k) \ \cdots \ \underline{d}^{(M)}(k)\right] \in \mathrm{C}^{P\times M}$ consists of M steering vectors, each given by

$$\underline{d}^{(m)}(k) \equiv \left[d_1^{(m)}(k) \ \cdots \ d_P^{(m)}(k)\right]^T \in \mathrm{C}^{P\times 1}$$

and

$$d_p^{(m)} \equiv a_p^{(m)} e^{-j2\pi t_p^{(m)}/N};$$

$\underline{S_0}(k) \equiv \left[S_0^{(1)}(k) \ \cdots \ S_0^{(M)}(k)\right]^T \in \mathrm{C}^{M\times 1}$ ($S_0^{(m)}(k)$ is the k^{th} DFT point of $s_0^{(m)}$, $m = 1,\cdots,M$).

The source signal spectra $\underline{S_0}(k)$ are unknown and deterministic. The noise spectral vector $\underline{U}(k) \in \mathrm{C}^{P\times 1}$ is a complex-valued zero-mean white Gaussian process, and each element has a variance $L\sigma^2$.

Here, the unknown parameter vector $\underline{\Theta} \in \mathrm{C}^{(MN+2M)\times 1}$ is

$$\underline{\Theta} \equiv \left[\underline{r_s}^T, \underline{S_0}^{(1)T} \ \cdots \ \underline{S_0}^{(m)T} \ \cdots \ \underline{S_0}^{(M)T}\right]^T,$$

where

$$\underline{r_s} \equiv \left[\underline{r_s}^{(1)T} \ \cdots \ \underline{r_s}^{(m)T} \ \cdots \ \underline{r_s}^{(M)T}\right]^T \in \mathrm{R}^{1\times 2M}$$

and

$$\underline{S_0}^{(m)} \equiv \left[S_0^{(m)}(0) \ \cdots \ S_0^{(m)}(N-1)\right]^T \in \mathrm{C}^{N\times 1}.$$

According to Eq. (2), we may construct the equivalent log-likelihood of $\underline{X}(k)$ after neglecting the constant terms, which is given by

$$\begin{aligned} J(\underline{r_s}) &= \log f_{\underline{X}}\left[\underline{\Theta}; \underline{X}(k)\right] \\ &\equiv -\sum_{k=1}^{N}\left[\underline{X}(k) - \overline{D}(k)\underline{S_0}(k)\right]^H \left[\underline{X}(k) - \overline{D}(k)\underline{S_0}(k)\right]. \end{aligned} \quad (3)$$

Thus, the maximum-likelihood estimation of $\underline{\Theta}$ can be achieved as

$$\begin{aligned} \hat{\underline{\Theta}} &= \arg\max\left(\log f_{\underline{X}}\left[\underline{\Theta}; \underline{X}(k)\right]\right) \\ &= \arg\min\left(\sum_{k=0}^{N-1}\left[\underline{X}(k) - \overline{D}(k)\underline{S_0}(k)\right]^H \left[\underline{X}(k) - \overline{D}(k)\underline{S_0}(k)\right]\right). \end{aligned} \quad (4)$$

Eq. (4) yields the source signal spectral estimates as

$$\hat{\underline{S}}_0(k) = (\overline{D}(k)^H \overline{D}(k))^{-1} \overline{D}(k)^H \underline{X}(k). \quad (5)$$

According to [1] and Eqs. (3), (5), the ML source location estimates can be obtained as

$$\arg\max\left(J(\underline{r_s})\right) = \arg\min\left(\sum_{k=0}^{N-1}\left\|\overline{P}(k,\underline{r_s})\underline{X}(k)\right\|^2\right), \quad (6)$$

where the *projection matrix* $\overline{P}(k,\underline{r_s})$ is defined as

$$\overline{P}(k,\underline{r_s}) \equiv \overline{D}(k)(\overline{D}(k)^H \overline{D}(k))^{-1} \overline{D}(k)^H. \quad (7)$$

The expectation maximization (EM) algorithm is a well-known iterative algorithm for the maximum-likelihood estimation [7]. The complicated nonlinear optimization problem in Eq. (6) can be simplified using the EM procedure incorporated with the augmented (complete) data corresponding to individual incident source signals. First, we denote the received signal spectrum (the k^{th} DFT point of $a_p^{(m)} s_0^{(m)}\left(n - t_p^{(m)}\right)$) as $X_p^{(m)}(k)$, $1 \le p \le P$, $1 \le m \le M, 0 \le k \le N-1$, from the m^{th} source to the p^{th} sensor. Then we define the augmented data as $\{\underline{X}^{(m)}(k); 1 \le m \le M, 0 \le k \le N-1\}$, where

$$\underline{X}^{(m)}(k) = [X_1^{(m)}(k),\ldots,X_P^{(m)}(k)]^T \in \mathrm{C}^{P\times 1}. \quad (8)$$

In addition, we assume that $\underline{X}^{(m)}(k)$ is a mixture Gaussian process with the cluster mean vectors $\underline{d}^{(m)}(k)S_0^{(m)}(k)$ and identical cluster covariance matrices $(1/M)\sigma^2 \overline{I}$ where $\overline{I}$ denotes the $P\times P$ identity matrix. The relationship between the observed (incomplete) data $\underline{X}(k)$ and the complete data is established as

$$\underline{X}(k) = \sum_{m=1}^{M} \underline{X}^{(m)}(k). \quad (9)$$

Since the complete data $\underline{X}^{(m)}(k)$ are not available, the expectation has to be performed using the current estimated parameters. Given the estimate $\underline{\Theta}^{[i]}$ for the i^{th} iteration, the

$(i+1)^{\text{th}}$ iteration of EM algorithm involves the following two steps:

Expectation (E-step)

Calculate

$$Q(\underline{\Theta}, \underline{\Theta}^{[i]}) = E\left\{\log f_{\underline{X}}\left[\underline{\Theta}; \hat{\underline{X}}^{(m)}(k, \underline{\Theta}^{[i]})\right]\right\}, \quad \text{for } m = 1, \cdots, M, \qquad (10)$$

where

$$\begin{aligned}\hat{\underline{X}}^{(m)}(k, \underline{\Theta}^{[i]}) &\equiv E\left[\underline{X}^{(m)}(k) \mid \underline{X}, \underline{\Theta}^{[i]}\right] \\ &= \underline{d}^{(m)}(k) S_0^{(m)}(k) + \frac{1}{M}(\underline{X}(k) - \overline{D}(k)\underline{S_0}(k)).\end{aligned} \qquad (11)$$

It is noted that $\underline{d}^{(m)}(k)$, $S_0^{(m)}(k)$, $\overline{D}(k)$, $\underline{S_0}(k)$, $k = 0, 1, \cdots, N-1$, are all estimated using $\underline{\hat{r}_s}^{(m)}$ obtained from the previous iteration i, according to the definitions below Eq. (2).

Maximization (M-step):

Re-estimate

$$\underline{\hat{r}_s}^{(m)} = \arg\max_{\underline{r_s}^{(m)}} \sum_{k=0}^{N-1} \left| \left[\underline{d}^{(m)}(k)\right]^H \hat{\underline{X}}^{(m)}(k, \underline{\Theta}^{[i]}) \right|^2, \qquad (12)$$

$$\hat{S}_0^{(m)}(k) = \frac{\left[\underline{d}^{(m)}(k)\right]^H \hat{\underline{X}}^{(m)}(k, \underline{\Theta}^{[i]})}{\left\| \underline{d}^{(m)}(k) \right\|^2}, \qquad (13)$$

$$\text{for } m = 1, \cdots, M, \; k = 0, 1, \cdots, N-1.$$

Then update

$$\underline{\Theta}^{[i+1]} = \left[\underline{\hat{r}_s}^T, \underline{\hat{S}_0}^{(1)T} \; \cdots \; \underline{\hat{S}_0}^{(m)T} \; \cdots \; \underline{\hat{S}_0}^{(M)T}\right]^T, \qquad (14)$$

where

$$\underline{\hat{r}_s} \equiv \left[\underline{\hat{r}_s}^{(1)T} \; \cdots \; \underline{\hat{r}_s}^{(m)T} \; \cdots \; \underline{\hat{r}_s}^{(M)T}\right]^T \qquad (15)$$

and

$$\underline{\hat{S}_0}^{(m)} \equiv \left[\hat{S}_0^{(m)}(0) \; \cdots \; \hat{S}_0^{(m)}(N-1)\right]^T, \qquad (16)$$

for $m = 1, \cdots, M$. The E- and M-steps are repeated until the pre-defined convergence of the estimated parameters is achieved.

III. Gaussianity Test for ML Source Localization

The ML estimation in Section II relies on the multivariate mixture Gaussian density model. However this assumption is not valid in general especially when the signal sample size is limited and the signal-to-noise (SNR) is large [8]. According to [1], the received signals at the sensor array are always modeled as a Gaussian mixture. Since the source spectrum $\underline{S_0}(k)$ is not necessarily Gaussian, the statistical mismatch would incur immediately.

The root-mean square location estimation error versus the comparative SNR (the average SNR over all the sensors) for different noise models are depicted in Figure 1. Interesting result can be found that the source localization performance is better when the uniform noise rather than Gaussian noise is added for the same comparative SNR. To explain this phenomenon, we employ the GT for the received signals in the two different kinds of ambient noise. Since the ML location estimation relies on the DFT, the Gaussianity measure has to be undertaken on the DFT sequences.

The received signal spectral waveform $X_p(k)$ at the p^{th} sensor is given by

$$\begin{aligned}X_p(k) &= \mathrm{Re}\{X_p(k)\} + \sqrt{-1}\,\mathrm{Im}\{X_p(k)\} \\ &= \sum_{n=0}^{N-1} \cos\left(\frac{2\pi kn}{N}\right) x(n) + \sqrt{-1} \sum_{n=0}^{N-1} \sin\left(\frac{2\pi kn}{N}\right) x(n),\end{aligned}$$

$$k = 0, 1, \cdots, N-1. \qquad (17)$$

According to Eq. (17), we may measure the Gaussianities separately for the real and imaginary parts of $X_p(k)$.

A. Edgeworth Expansion for PDF Characterization

As previously mentioned, the finite sample size and high SNR will lead to the non-Gaussian characteristics of the received signals [8]. The probability density function (PDF) mismatch between the underlying Gaussian model and the actual statistics can be evaluated using the Edgeworth expansion. Similar to the Gram-Charlier series, the Edgeworth expansion can be used to characterize the unknown PDF based on the moments and the cumulants.

The Edgeworth expansion of an arbitrary probability density function $f_x(x)$ can be written as [10]:

$$f_x(x) = \vartheta(x)\left\{1 + \sum_{k=2}^{+\infty} P_x(x)\right\}, \qquad (18)$$

where $\vartheta(x)$ is the zero-mean univariate Gaussian PDF, which is given by

$$\vartheta(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right). \qquad (19)$$

The polynomial $P_x(x)$ is defined as

$$P_k(x) \equiv \sum_{m=1}^{k-2} \frac{1}{m!}\left(\sum_{j_1+j_2+\cdots j_m} \frac{\chi_{j_1+2}\chi_{j_2+2}\cdots\cdots\chi_{j_m+2}}{(j_1+2)!(j_2+2)!\cdots\cdots(j_m+2)} H_{k+2m-2}(x)\right), \qquad (20)$$

where χ_j is the j^{th}-order cumulant of x, which is defined as:

$$\chi_j = (-1)^j \frac{d^j}{d\eta^j} \log \hat{f}_x(\eta)\Bigg|_{\eta=0}. \qquad (21)$$

$\hat{f}_x(\eta)$ is the characteristic function of the random variable x and $H_j(x)$ is the j^{th}-order Hermite polynomial such that

$$\vartheta(x)H_j(x) = (-1)^j \frac{d^j}{d\eta^j}\vartheta(x) . \quad (22)$$

B. Gaussianity Measure Using Bispectrum

The Gaussianity measure based on the bispectrum was used to examine the statistics of time series [15]. If $x(0), x(1), \ldots, x(N-1)$ is the sensor signal, its bispectrum is defined as

$$\hat{C}_{xxx}(i', i'') \equiv N^{-1} X(i') X(i'') X^*(i' + i'') , \quad (23)$$

where $X(i')$ is the N-point discrete Fourier transform of the signal. The estimated bispectrum is smoothed by a two-dimensional window $W(i', i'')$ (window size is $\mathsf{M} \times \mathsf{M}$). Then a sampled bispectrum is used to construct a statistics to test whether the bispectrum given by Eq. (23) is nonzero; a rejection action of the null hypothesis implies that the signal is non-Gaussian [15]. The statistics is constructed below. According to Eq. (23), we can compute

$$\varsigma_{i',i''} = \frac{\hat{C}_{xxx}(i', i'')}{[N / \mathsf{M}^2]^{1/2} [\hat{S}_{xx}(i') \hat{S}_{xx}(i'') \hat{S}_{xx}(i' + i'')]^{1/2}} . \quad (24)$$

It can be proved that the PDF of $\varsigma_{i',i''}$ is complex Gaussian with unit-variance. Here $\hat{S}_{xx}(i')$ is the sample estimate for the power spectrum of x. Consequently, $\left|\varsigma_{i',i''}\right|^2$ is approximately a chi-square random variable with two degrees of freedom. Thus, we can construct the statistics Φ for the Gaussianity test such that

$$\Phi = 2 \sum_{i'} \sum_{i''} \left| \varsigma_{i',i''} \right|^2 . \quad (25)$$

Asymptotically speaking, Φ is chi-square distributed under the null hypothesis of Gaussianity. Hence it is easy to derive a statistical test to determine whether the observation is consistent with a central chi-squared distribution; this "consistency" is characterized as the probability-of-false-alarm value, that is, the probability that the sensor data possess a nonzero bispectrum. If this probability-of-false-alarm value is small, we can accept the Gaussian assumption.

Since the sample size is limited in source localization, we can not directly apply the technique in [15] (it requires a large sample size) to estimate the bispectrum of the sensor signal. Instead, we use the bootstrap algorithm which is more appropriate for finite sample sizes [18]. The estimation result within a primary region D is considered only due to the symmetry of the bispectrum such that

$$D \equiv \left\{ 0 < j \le \frac{N}{2}, 0 < k < j, 2j + k < N \right\} . \quad (26)$$

We propose to use this Gaussianity measure for the robustness analysis of the ML source localization. This new analysis can be manifested in the next section.

IV. SIMULATION

To demonstrate how to employ our proposed Gaussianity test or measure described in Section III, we present the simulation results here. An acoustic source signal was acquired from [1]. The sampling frequency is 100 kHz. The propagation speed is 345 meters/sec. The data is simulated for a circularly-shaped array of five sensors using the aforementioned acoustic data. The detailed setup can be referred to [7]. The sample size is L = 200, and the DFT size is N = 256.

Fifty Monte Carlo experiments are carried out using randomly initiated source location estimates at different signal-to-noise ratios. Two sources are considered in this simulation. The EM algorithm in [7] is employed for the source localization in this simulation.

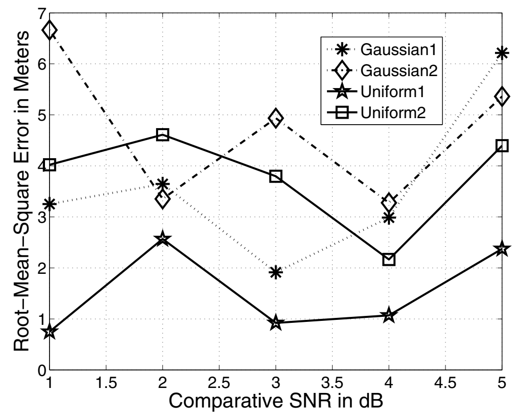

Figure 1. Root-mean-square error versus comparative SNR for different noise models (Gaussian1: source 1 is added with Gaussian nose, Gaussian2: source 2 is added with Gaussian nose, Uniform1: source 1 is added with uniform nose, Uniform2: source 2 is added with uniform nose). The comparative SNR is the average SNR over all the sensor signals.

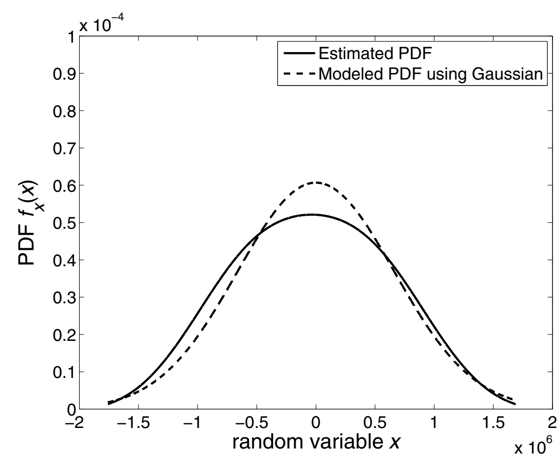

Figure 2. The estimated PDF (using the Edgeworth expansion) and the modeled PDF using Gaussian distribution for the finite data generated by the computer.

To illustrate the Gaussianity test, we generate a data set, namely the acoustic source signal embedded in the additive Gaussian noise. Two PDF estimators are employed. First, we use the Edgeworth expansion to estimate the PDF (illustrated as "estimated PDF" in Figure 2); second, we use the underlying Gaussian PDF to model the statistics of the data (illustrated as "Modeled PDF using Gaussian"). Figure 3 depicts the estimated bispectrum for the aforementioned data set using the bootstrap algorithm stated in Section III.

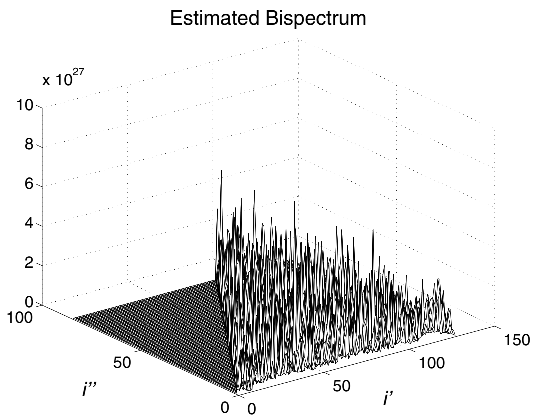

Figure 3. The estimated bispectrum $\left|\hat{C}_{xxx}(i',i'')\right|$.

To demonstrate the robustness analysis, we apply the rejection Gaussian hypothesis in [15] for the two sensor signal sets used to generate Figure 1. Figure 4 depicts the probabilities of rejection (as stated in Section III.B) for the aforementioned data added with either Gaussian (denoted as "with Gaussian noise" in the figure) or uniform noise (denoted as "with uniform noise" in the figure). According to Figure 4, the interesting result can be found that the sensor data involving the uniform noise is "more Gaussian" than that involving the Gaussian noise.

V. CONCLUSION

In this paper, we derive the robustness analysis for the maximum-likelihood source localization. The robustness is majorly affected by the actual statistics of the sensor data. Using the Edgeworth expansion and the bispectrum, we can measure the departure of Gaussianity for different sensor signals or the received signals with different kinds of ambient noise. Using our newly derived Gaussianity test for the sensor signals, we can quantify the statistical mismatch and provide the robustness figures which can be utilized to predict the localization performance comparison for different received signals.

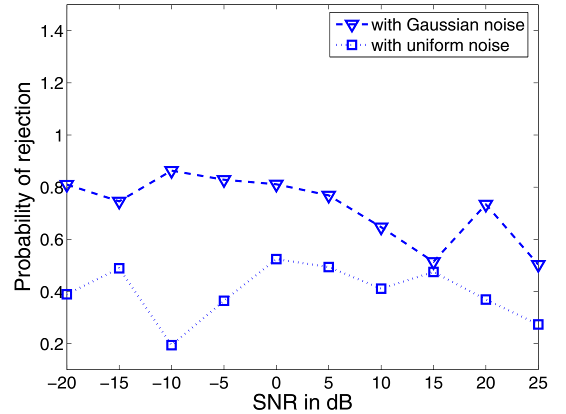

Figure 4. Probability of rejection versus SNR.

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