COT 5407 Introduction to Algorithms
Homework 2

Due *in class* on Tuesday, October 02, 2018

This second homework covers Ch 6, 7, 8, 9, and sections 5.1 and 5.2 of the book

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1. **[5 points]**

   Using Figure 6.2 (page 155) as a model, illustrate the operation of MAX-HEAPIFY(A,3) on the array $A =< 30, 18, 3, 16, 14, 11, 1, 5, 27, 11, 4, 8, 9, 10 >$

2. **[5 points]** Using Figure 6.2 (page 155) as a model, illustrate the operation of HEAPSORT on the array $A =< 16, 3, 12, 26, 5, 4, 22, 8, 14 >$

3. [5 points] Using Figure 8.2 (page 195) as a model, illustrate the operation of COUNTING-SORT on the array $A =< 5, 0, 2, 0, 1, 4, 3, 5, 3, 2, 2 >$

4. **[5 points]** Using Figure 8.3 as a model (page 198), illustrate the operation of RADIX-SORT on the following list of English words: ROB,COW, BOT, DOG, RUG, ROW, MOB, BOX, TAB, BAT, EAR, TAR, DIG, BIG, BAT, NOW, FOX

5. **[5 points]** Solve exercise 9.1-1 in Cormen

6. **[5 points]** Solve exercise 9.2-4 in Cormen

7. **[10 points]** Carefully read sections 5.1 and 5.2 of the book and solve the following problem:

   A hat-check worker completely loses track of which of n hats belong to which owners and gives the hats back to the owners in a random order. What is the expected number of owners that get back their own hat? Use indicator random variables to solve the problem.

   For the reading part do not write anything in your solution.

8. **[10 points]**

   Describe an algorithm that implements the operation HEAP-REMOVAL(A, i) which erases the item in the node i from the heap A. Your algorithm should run in time $O(lgn)$ for a max-heap with n-elements.

9. **[10 points]**

   Show that when an A array is sorted in decreasing order and it contains distinct elements the running time of QUICKSORT is $\Theta(n^2)$.

10. **[10 points] Radix Sort**

    - Using induction sort show that radix sorts works. Does the intermediate sort need to be stable? Justify your answer.
    - For the following sorting algorithms write down if the are stable: insertion sort, merge sort, heapsort, and quicksort. Write a small justification.

11. **[15 points]** For an array of n distinct elements $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, ..., w_n$ s.t $\sum_{i=1}^{n} w_i = 1$, the weighted median is the element x_k that satisfies $\sum_{x_i < x_k} w_i < \frac{1}{2}$ and $\sum_{x_i > x_k} w_i \leq \frac{1}{2}$.

    - Show that the median of $x_1, x_2, \ldots, x_n$ is the weighted median of $x_1, x_2, \ldots, x_n$ when the weights are $w_i = 1/n$ for $i = 1, 2, \ldots, n$
    - Propose an algorithm that computes the weighted median of an array with n elements in $O(nlgn)$ worst-case (Hint: use sorting).
    - Describe and analyze an algorithm to compute the weighted median of a given weighted set in $O(n)$ time (Hint: Carefully read and understand section 9.3).

12. **[15 points]** Carefully read page 171-173 from Cormen 3rd edition where the correctness of PARTITION in quick sort is presented. Then, solve problem **7-1** (pg 185). For the reading part do not write anything in your solution.