Fault-Tolerant Spanners
(ABSTRACT)

Christos Levcopoulos[*] Giri Narasimhan[†] Michiel Smid[‡]

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Spanners have applications in the design of geometric networks. Given S, a set of n points in d-dimensional Euclidean space, a Euclidean t-spanner is a graph G in which for every pair of vertices $x, y \in S$, there exists a path in G of length at most t times the Euclidean distance between x and y. Previous results [?] have shown that for a set of n points in d-dimensional space, t-spanners (for any constant $t > 1$) with many nice properties, including small total length (at most a constant times the length of a minimum spanning tree) and bounded degree, can be computed in $O(n \lg n)$ time.

We show that it is possible to incorporate fault tolerance into such networks. Fault tolerance is intimately related to the graph-theoretic concept of *connectivity*. Vertex (edge) connectivity is defined as the number of vertices (edges) that can be removed without disconnecting the graph. Fault-tolerant networks are usually designed by making them highly connected. We construct networks that are more than just resilient to vertex or edge faults. In the following, let k be a fixed constant bounding the number of faults in the network.

First, we give a simple construction of a network (based on a local replacement strategy) that displays several interesting properties relating to fault tolerance.

Next, we give an alternate construction of a network $G = (S, E)$, based on well-separated pairs, such that for any set $E' \subset E$ of size at most k, and any pair of vertices, $x, y \in S$, the graph $G \setminus E'$ contains an xy-path having

[*]Department of Computer Science, Lund University, Sweden; *e-mail*: Christos.Levcopoulos@dna.lth.se

[†]Dept. of Mathematical Sciences, The University of Memphis, Memphis TN 38152; Research Supported by NSF Grant CCR-940-9752; *e-mail*: giri@msci.memphis.edu

[‡]Otto-von-Guericke-Universität, Institut für Simulation und Graphik, Postfach 4120, 39016 Magdeburg, Germany; *e-mail*: michiel@isgnw.cs.uni-magdeburg.de

length at most t times the length of the shortest path in the graph $K_S \setminus E'$. Here, K_S denotes the complete Euclidean graph on the points of S. This network G can be constructed in $O(n \log^{d-1} n)$ time. A similar result is shown for vertex faults.

We then show how to prune the above network G, thereby obtaining a subnetwork G', for which the following holds. For any set $E' \subset E$ of size at most k, the network $G' \setminus E'$ has the property that, for any pair of vertices, $x, y \in S$, there exists a xy-path of length at most a constant times the length of the shortest path in $K_s \setminus E'$. Again, a similar result holds for vertex faults. Moreover, the degree of this network is bounded by a constant, and its total length is proportional to the weight of a minimum spanning tree of S.

Thus, our networks guarantee the existence of reasonably short alternate paths in the presence of faults without increasing the total length of the network or the maximum degree of the vertices by more than a constant factor.

One of the ingredients of our algorithms is an efficient data structure for the following query problem. Given two arbitrary query points x and y, compute a point $p \in S$, such that $|xp| + |py|$ is minimal. For our purposes, it suffices to find an approximation. Using range trees, we can find such an approximation in $O(\lg^d n)$ time; moreover, a batch of $O(n)$ queries can be solved in $O(n \lg^{d-1} n)$ total time. We also show that the exact version of this problem can be solved using parametric search and linearization in close to $O(\sqrt{n})$ time per query.

References

[ADM+95] S. Arya, G. Das, D. M. Mount, J. S. Salowe, and M. Smid. Euclidean spanners: short, thin, and lanky. In *Proc. 27th Annu. ACM Sympos. Theory Comput.*, pages 489–498, 1995.