COP 3530: Fall 2016: Quiz 1 Solutions

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1. [2]

$$\sum_{i=1}^{N}\sum_{j=i}^{1} 1 = \sum_{i=1}^{N}\sum_{j=1}^{i} 1 = \sum_{i=1}^{N} i = N(N+1)/2 = O(N^2)$$

2. [1] The definition is as follows: **We say that $f(n) = O(g(n))$, if there exists positive constants c and n_0 such that $f(n) \leq cg(n)$, for all $n \geq n_0$.** Thus the right answer is (b).

3. [1] The above definition also means that in order to disprove $f(n) = O(g(n))$, we need to show that for all possible positive constants c and n_0, there is some value of $n \geq n_0$, for which $f(n) > cg(n)$. As discussed in class, it is not possible to try every possible value of the constants c and n_0. Thus, the right answer is (d).

4. [3] Intuition tells us that $16n = O(n^3)$ must be true because n^3 has a higher exponent in its dominant term than $16n$.

   In order to prove this, we know that we have to find constants c and n_0 that will satisfy the above definition.

   If we choose $c = 1$, then we will need $n_0 = 4$ in order for $16n \leq cn^3 = n^3$ to be satisfied.

   On the other hand, if we choose $c = 4$, then $n_0 = 2$ is sufficient for $16n \leq cn^3 = 4n^3$ to be satisfied.

   Note that $16n = O(n^3)$ also means that $n^3 = \omega(16n)$.

5. [3] Using our intuition from above, we know that we should try to disprove the claim that: $n^3 = O(16n)$. As you know, our approach is to employ a proof by contradiction.

   Now assume that the statement $n^3 = O(16n)$ is true.

   By the definition, we know that there exists positive constants c and n_0 such that $n^3 \leq 16cn$, for all $n \geq n_0$.

   Since this could be achieved with many different values of c and n_0, we will assume that we arbitrarily fix it to one of those pairs of values.

   If $n^3 \leq 16cn$, for all $n \geq n_0$, we know that

   $n^2 \leq 16c$, for all $n \geq n_0$.

   However, if we choose any value of $n \geq \max n_0, 4\sqrt{c}$, then the above statement is contradicted.

   Thus, our assumption must be wrong. Hence $n^3 \neq O(16n)$.