Binary Search Trees

Values in the left subtree are smaller than the value stored at root.
Values in the right subtree are larger than the value stored at root.

Figure 19.1

Two binary trees: (a) a search tree;
(b) not a search tree

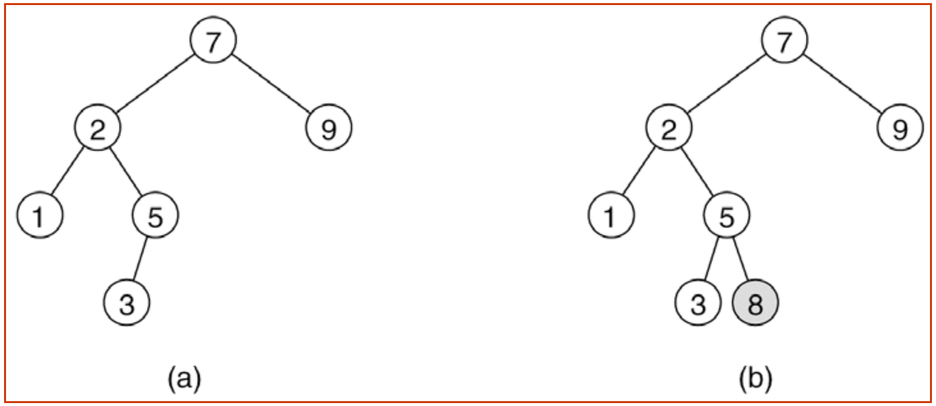


Figure 19.1

Two binary trees: (a) a search tree;
(b) not a search tree

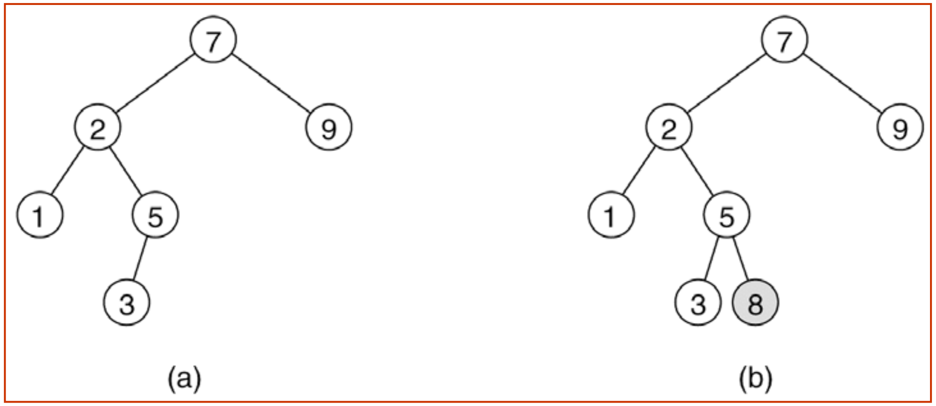


Figure 19.2

Binary search trees
(a) before and (b) after the insertion of 6

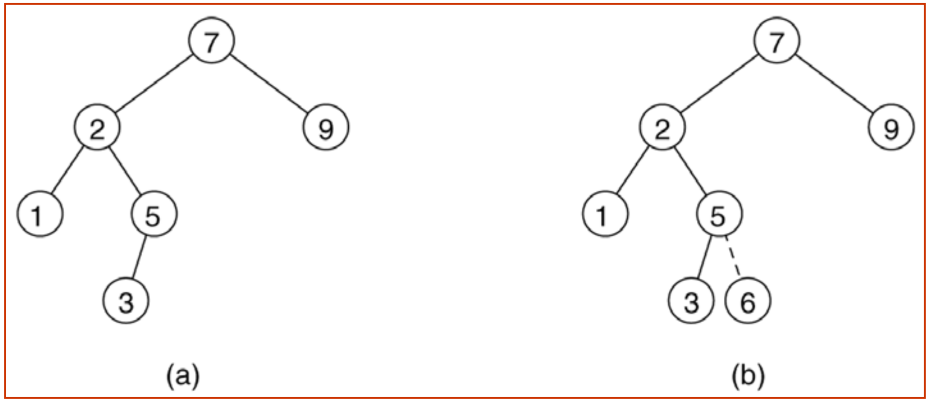


Figure 19.3

Deletion of node 5 with one child:
(a) before and (b) after

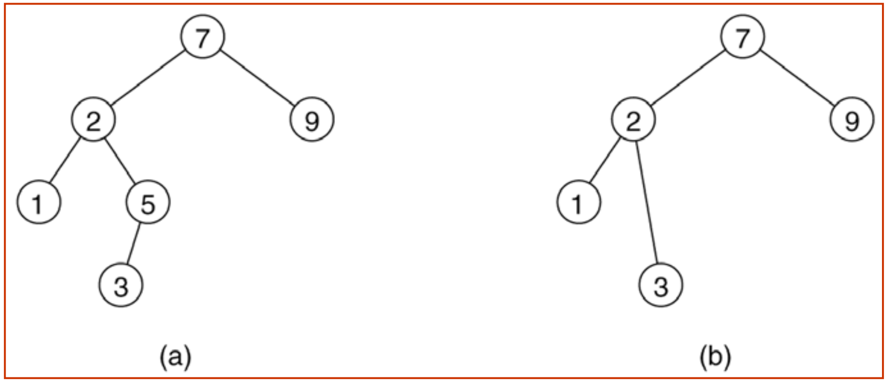
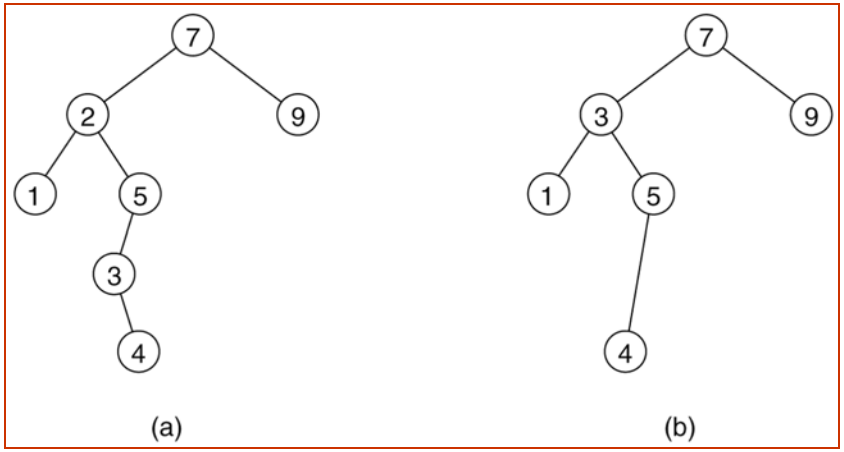


Figure 19.4

Deletion of node 2 with two children:
(a) before and (b) after



Data Structures & Problem Solving using JAVA/2E Mark Allen Weiss © 2002 Addison Wesley

Binary Search Trees

```
// BinarySearchTree class
// void insert( x )       --> Insert x
// void remove( x )       --> Remove x
// void removeMin( )      --> Remove minimum item
// Comparable find( x )   --> Return item that matches x
// Comparable findMin( ) / findMax( ) --> Return smallest / largest item
// boolean isEmpty( )     --> Return true if empty; else false
// void makeEmpty( )      --> Remove all items
public class BinarySearchTree
{  private Comparable elementAt( BinaryNode t ) { return t == null ? null :
      t.element; }
   protected BinaryNode insert( Comparable x, BinaryNode t )
   protected BinaryNode remove( Comparable x, BinaryNode t )
   protected BinaryNode removeMin( BinaryNode t )
   protected BinaryNode findMin( BinaryNode t )
   private BinaryNode findMax( BinaryNode t )
   private BinaryNode find( Comparable x, BinaryNode t )

  protected BinaryNode root;
}
```

Binary Search Trees

```
public static void main( String [ ] args ) {
        BinarySearchTree t = new BinarySearchTree( );
        final int NUMS = 4000;
        final int GAP  =   37;
        System.out.println( "Checking... (no more output means success)" );
        for( int i = GAP; i != 0; i = ( i + GAP ) % NUMS )
            t.insert( new Integer( i ) );
        for( int i = 1; i < NUMS; i+= 2 )
            t.remove( new Integer( i ) );
        if( ((Integer)(t.findMin( ))).intValue( ) != 2 ||
            ((Integer)(t.findMax( ))).intValue( ) != NUMS - 2 )
            System.out.println( "FindMin or FindMax error!" );
        for( int i = 2; i < NUMS; i+=2 )
             if( ((Integer)(t.find( new Integer( i ) ))).intValue( ) != i )
                 System.out.println( "Find error1!" );
        for( int i = 1; i < NUMS; i+=2 )
        {
            if( t.find( new Integer( i ) ) != null )
                System.out.println( "Find error2!" );
        }
    }
}
```

Binary Search Trees

```
protected BinaryNode insert( Comparable x, BinaryNode t )   {
        if( t == null )
            t = new BinaryNode( x );
        else if( x.compareTo( t.element ) < 0 )
            t.left = insert( x, t.left );
        else if( x.compareTo( t.element ) > 0 )
            t.right = insert( x, t.right );
        else   throw new DuplicateItemException( x.toString( ) );  // Duplicate
        return t;
    }
protected BinaryNode remove( Comparable x, BinaryNode t )  {
        if( t == null )   throw new ItemNotFoundException( x.toString( ) );
        if( x.compareTo( t.element ) < 0 )    t.left = remove( x, t.left );
        else if( x.compareTo( t.element ) > 0 )    t.right = remove( x, t.right );
        else if( t.left != null && t.right != null )    {
            t.element = findMin( t.right ).element;
            t.right = removeMin( t.right );
        }
        else
            t = ( t.left != null ) ? t.left : t.right;
        return t;
    }
```

Binary Search Trees

```
protected BinaryNode removeMin( BinaryNode t )
  {
    if( t == null )
      throw new ItemNotFoundException( );
    else if( t.left != null )
    {
      t.left = removeMin( t.left );
      return t;
    }
    else
      return t.right;
  }
```

```
protected BinaryNode findMin( BinaryNode t )
  {
    if( t != null )
      while( t.left != null )
        t = t.left;
    return t;
  }

private BinaryNode
            find( Comparable x, BinaryNode t )
  {
    while( t != null )
    {
      if( x.compareTo( t.element ) < 0 )
        t = t.left;
      else if( x.compareTo( t.element ) > 0 )
        t = t.right;
      else
        return t;    // Match
    }
    return null;       // Not found
  }
```

Figure 8.3

Basic action of insertion sort (the shaded part is sorted)

Array Position	0	1	2	3	4	5
Initial State	8	5	9	2	6	3
After a[0..1] is sorted	5	8	9	2	6	3
After a[0..2] is sorted	5	8	9	2	6	3
After a[0..3] is sorted	2	5	8	9	6	3
After a[0..4] is sorted	2	5	6	8	9	3
After a[0..5] is sorted	2	3	5	6	8	9

Data Structures & Problem Solving using JAVA/2E Mark Allen Weiss © 2002 Addison Wesley

Figure 8.4

A closer look at the action of insertion sort (the dark shading indicates the sorted area; the light shading is where the new element was placed).

Array Position	0	1	2	3	4	5
Initial State	8	**5**				
After a[0..1] is sorted	5	8	**9**			
After a[0..2] is sorted	5	8	9	**2**		
After a[0..3] is sorted	2	5	8	9	**6**	
After a[0..4] is sorted	2	5	6	8	9	**3**
After a[0..5] is sorted	2	3	5	6	8	9

Data Structures & Problem Solving using JAVA/2E Mark Allen Weiss © 2002 Addison Wesley

Insertion Sort

```
public static void insertionSort( Comparable [ ] a )
  {
    for( int p = 1; p < a.length; p++ )
    {
      Comparable tmp = a[ p ];
      int j = p;

      for( ; j > 0 && tmp.compareTo( a[ j - 1 ] ) < 0; j-- )
        a[ j ] = a[ j - 1 ];
      a[ j ] = tmp;
    }
  }
```

Figure 8.5

Shellsort after each pass if the increment sequence is {1, 3, 5}

ORIGINAL	81	94	11	96	12	35	17	95	28	58	41	75	15
After 5-sort	35	17	11	28	12	41	75	15	96	58	81	94	95
After 3-sort	28	12	11	35	15	41	58	17	94	75	81	96	95
After 1-sort	11	12	15	17	28	35	41	58	75	81	94	95	96

 © 2002 Addison Wesley

ShellSort

```
public static void shellsort( Comparable [ ] a )
  {
      for( int gap = a.length / 2; gap > 0;
                   gap = gap == 2 ? 1 : (int) ( gap / 2.2 ) )
         for( int i = gap; i < a.length; i++ )
         {
             Comparable tmp = a[ i ];
             int j = i;

             for( ; j >= gap && tmp.compareTo( a[ j - gap ] ) < 0; j -= gap )
                 a[ j ] = a[ j - gap ];
             a[ j ] = tmp;
         }
  }
```

Merge Sort

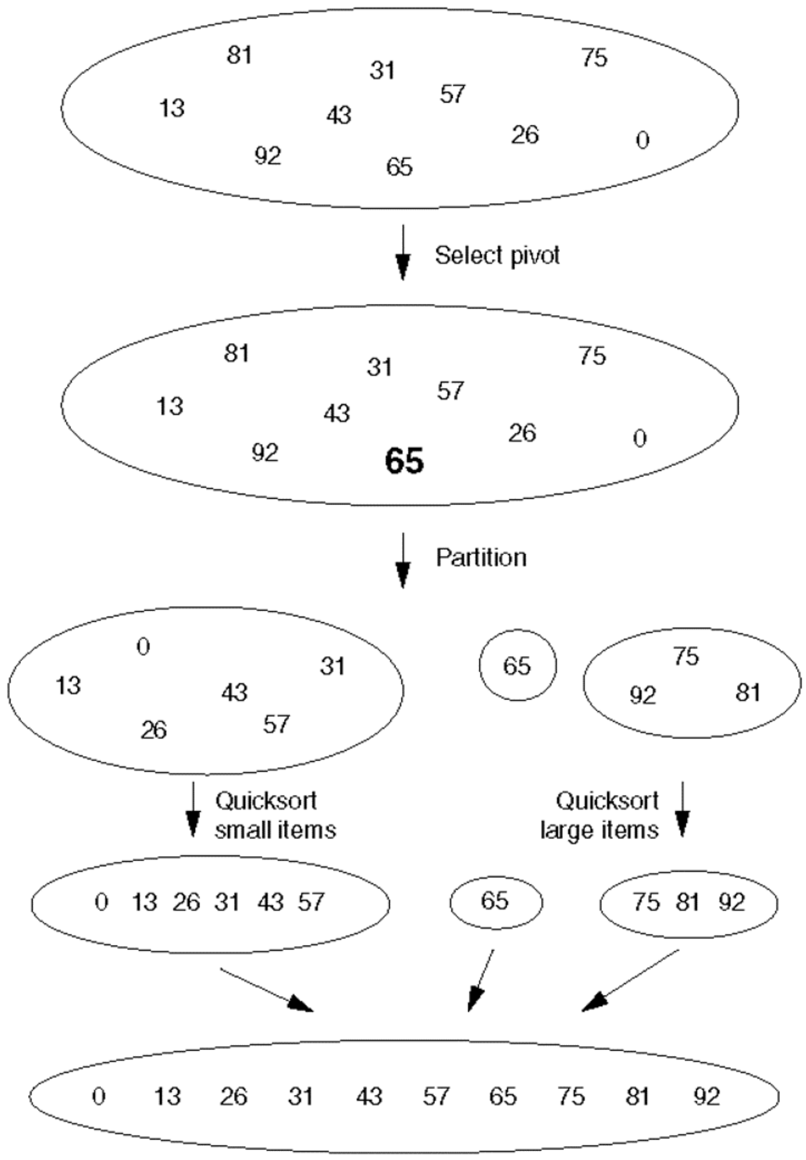
```
public static void mergeSort( Comparable [ ] a ) {
      Comparable [ ] tmpArray = new Comparable[ a.length ];
      mergeSort( a, tmpArray, 0, a.length - 1 );
  }
private static void mergeSort( Comparable [ ] a, Comparable [ ]
   tmpArray,
            int left, int right )
  {
      if( left < right )
      {
          int center = ( left + right ) / 2;
          mergeSort( a, tmpArray, left, center );
          mergeSort( a, tmpArray, center + 1, right );
          merge( a, tmpArray, left, center + 1, right );
      }
  }
```

Merge in Merge Sort

```
private static void merge( Comparable [ ] a, Comparable [ ] tmpArray,
                          int leftPos, int rightPos, int rightEnd )
  {
     int leftEnd = rightPos - 1;
     int tmpPos = leftPos;
     int numElements = rightEnd - leftPos + 1;
     while( leftPos <= leftEnd && rightPos <= rightEnd )
        if( a[ leftPos ].compareTo( a[ rightPos ] ) < 0 )
                tmpArray[ tmpPos++ ] = a[ leftPos++ ];
        else
                tmpArray[ tmpPos++ ] = a[ rightPos++ ];
     while( leftPos <= leftEnd )    // Copy rest of first half
        tmpArray[ tmpPos++ ] = a[ leftPos++ ];
     while( rightPos <= rightEnd )  // Copy rest of right half
        tmpArray[ tmpPos++ ] = a[ rightPos++ ];

    for( int i = 0; i < numElements; i++, rightEnd-- )
        a[ rightEnd ] = tmpArray[ rightEnd ];
  }
```

Figure 8.10 Quicksort



Data Structures & Problem Solving using JAVA/2E Mark Allen Weiss © 2002 Addison Wesley

Figure 8.11 Partitioning algorithm: Pivot element 6 is placed at the end.

8	1	4	9	0	3	5	2	7	6

Figure 8.12 Partitioning algorithm: i stops at large element 8; j stops at small element 2.

8	1	4	9	0	3	5	2	7	6

Figure 8.13 Partitioning algorithm: The out-of-order elements 8 and 2 are swapped.

2	1	4	9	0	3	5	8	7	6

Figure 8.14 Partitioning algorithm: i stops at large element 9; j stops at small element 5.

2	1	4	9	0	3	5	8	7	6

Figure 8.15 Partitioning algorithm: The out-of-order elements 9 and 5 are swapped.

2	1	4	5	0	3	9	8	7	6

Figure 8.16 Partitioning algorithm: i stops at large element 9; j stops at small element 3.

2	1	4	5	0	3	9	8	7	6

Figure 8.17 Partitioning algorithm: Swap pivot and element in position i.

2	1	4	5	0	3	6	8	7	9

Figure 8.18 Original array

8	1	4	9	6	3	5	2	7	0

Figure 8.19 Result of sorting three elements (first, middle, and last)

0	1	4	9	6	3	5	2	7	8

Figure 8.20 Result of swapping the pivot with the next-to-last element

0	1	4	9	7	3	5	2	6	8

Data Structures & Problem Solving using JAVA/2E Mark Allen Weiss © 2002 Addison Wesley

Quicksort

```
public static void quicksort( Comparable [ ] a ) {  quicksort( a, 0, a.length - 1 );  }
private static void quicksort( Comparable [ ] a, int low, int high )
  {
     if( low + CUTOFF > high )  insertionSort( a, low, high );
     else {  // Sort low, middle, high
        int middle = ( low + high ) / 2;
        if( a[ middle ].compareTo( a[ low ] ) < 0 )    swapReferences( a, low, middle );
        if( a[ high ].compareTo( a[ low ] ) < 0 )       swapReferences( a, low, high );
        if( a[ high ].compareTo( a[ middle ] ) < 0 )   swapReferences( a, middle, high );
        swapReferences( a, middle, high - 1 );         // Place pivot at position high - 1
        Comparable pivot = a[ high - 1 ];
        int i, j; // Begin partitioning
        for( i = low, j = high - 1; ; ) {
           while( a[ ++i ].compareTo( pivot ) < 0 ) /* Do nothing */     ;
           while( pivot.compareTo( a[ --j ] ) < 0 ) /* Do nothing */     ;
           if( i >= j )    break;
           swapReferences( a, i, j );
        }
       swapReferences( a, i, high - 1 );
        quicksort( a, low, i - 1 );    // Sort small elements
        quicksort( a, i + 1, high );   // Sort large elements
     }
  }
```