Evaluation

- Exams (2) 50%
- Homework Assignments 35%
- Semester Project 10%
- Class Participation 5%

Celebrity Problem

- A **Celebrity** is one that knows nobody and that everybody knows.

Celebrity Problem:

INPUT**:** n persons with a n×n information matrix.

OUTPUT**:** Find the "celebrity", if one exists.

MODEL**:** Only allowable questions are:

- ***Does person* i *know person* j*?***

- Naive Algorithm: $O(n^2)$ Questions.
- Using Divide-and-Conquer: $O(n \log_2 n)$ Questions.
- Improved solution?

Celebrity Problem (Cont'd)

- Naive Algorithm: $O(n^2)$ Questions.
  - Ask everyone of everyone else for a total of n(n-1) questions
- Using Divide-and-Conquer: $O(n \log_2 n)$ Questions.
  - Divide the people into two equal sets. Solve recursively and find two candidate celebrities from the two halves. Then verify which one (if any) is a celebrity by asking n-1 questions to each of them and n-1 questions to everyone else about them. This gives a recurrence for the total number of questions asked: $T(n) = 2T(n/2) + 2n$
- Improved solution?
  - Hint: What information do you gain by asking one question?

Celebrity Problem (Cont'd)

- **Induction Hypothesis 2:** We know how to find n-2 non-celebrities among a set of n-1 people, i.e., we know how to find at most one person among a set of n-1 people that could potentially be a celebrity.
- Resulting algorithm needs [3(n-1)-1] questions.

Solving Recurrence Relations

Page 62, [CLR]

Recurrence; Cond	Solution
$T(n) = T(n-1) + O(1)$	$T(n) = O(n)$
$T(n) = T(n-1) + O(n)$	$T(n) = O(n^2)$
$T(n) = T(n-c) + O(1)$	$T(n) = O(n)$
$T(n) = T(n-c) + O(n)$	$T(n) = O(n^2)$
$T(n) = 2T(n/2) + O(n)$	$T(n) = O(n \log n)$
$T(n) = aT(n/b) + O(n);$ $a = b$	$T(n) = O(n \log n)$
$T(n) = aT(n/b) + O(n);$ $a < b$	$T(n) = O(n)$
$T(n) = aT(n/b) + f(n);$ $f(n) = O(n^{\log_b a - \epsilon})$	$T(n) = O(n)$
$T(n) = aT(n/b) + f(n);$ $f(n) = O(n^{\log_b a})$	$T(n) = \Theta(n^{\log_b a} \log n)$
$T(n) = aT(n/b) + f(n);$ $f(n) = \Theta(f(n))$ $af(n/b) \leq cf(n)$	$T(n) = \Omega(n^{\log_b a} \log n)$

5