Animation Demos

http://www-cse.uta.edu/~holder/courses/cse2320/lectures/applets/sort1/heapsort.html

http://cg.scs.carleton.ca/~morin/misc/sortalg/

Order Statistics

- Maximum, Minimum $n-1$ comparisons

7	3	1	9	4	8	2	5	0	6

- MinMax
  - $2(n-1)$ comparisons
  - $3n/2$ comparisons
- Max and 2ndMax
  - $(n-1) + (n-2)$ comparisons
  - ???

k-Selection; Median

- Select the k-th smallest item in list
- Naïve Solution
  - Sort;
  - pick the k-th smallest item in sorted list.

    $O(n \log n)$ time complexity
- Randomized solution: Average case $O(n)$
- Improved Solution: worst case $O(n)$

```
QuickSort(A, p, r)
    if (p < r) then
        q = Partition(A, p, r)
        QuickSort(A, p, q)
        QuickSort(A, q+1, r)

Partition(A, p, r)
  x = A[r]
  i = p-1
  for j = p to r-1 do
      if (A[j] <= x) then
          i++
          SWAP(A[i], A[j])
  SWAP(A[i+1], A[r])
  return i+1
```

Partition Procedure Revisited

- The Partition code can be rewritten so that it accepts another parameter, namely, the pivot value. Let's call this new variation as PivotPartition.
- This change does not affect its time complexity.
- RandomizedPartition as used in RandomizedSelect picks the pivot uniformly at random from among the elements in the list to be partitioned.

Randomized Selection

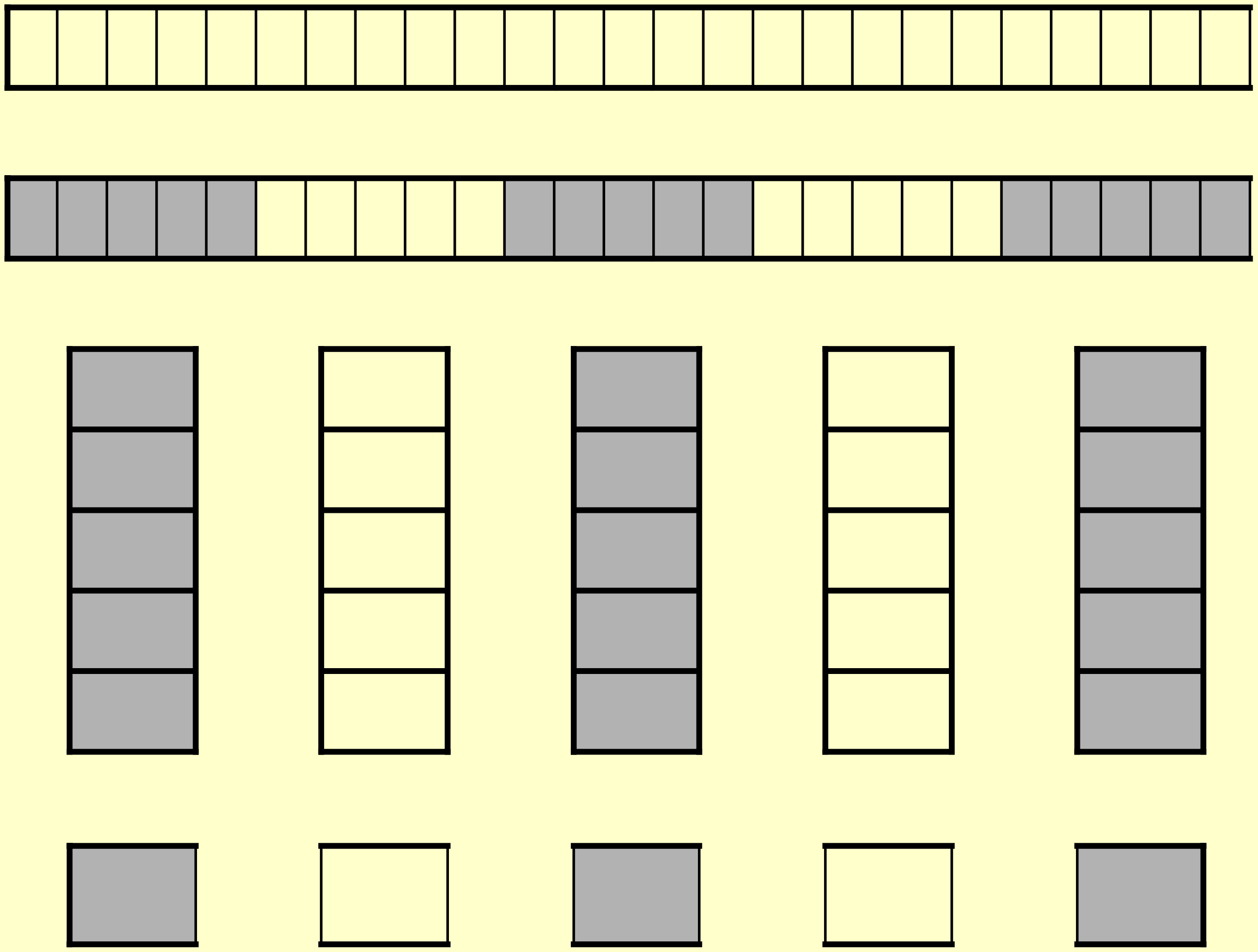
```
**RandomizedSelect**(A, p, r, i)
    if (p = r) then
        return A[p]
    q = RandomizedPartition(A, p, r)
    k = q – p + 1
    if (i = k)
        return A[i]
   else if (i < k)
        return RandomizedSelect(A, p, q-1, i)
    else
        return RandomizedSelect(A, q+1, r, i-k)
```

Randomized Selection: Rewritten

```
RandomizedSelect(A, p, r, i)
    if (p = r) then
        return A[p]
    Pivot = A[random(p,r)]
    q = PivotPartition(A, p, r, Pivot)
    k = q – p + 1
    if (i = k)
        return A[i]
    else if (i < k)
        return RandomizedSelect(A, p, q-1, i)
    else
        return RandomizedSelect(A, q+1, r, i-k)
```

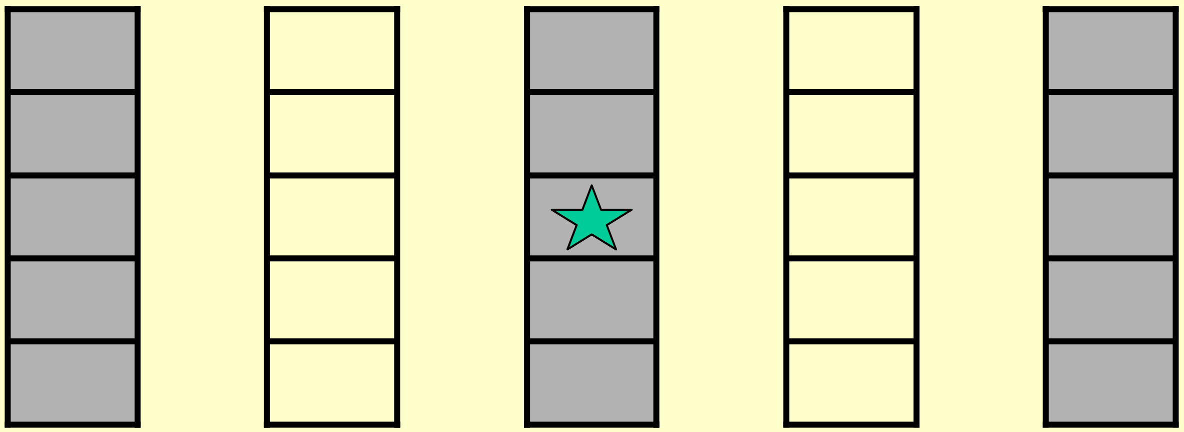
k-Selection & Median: Improved Algorithm

- Start with initial array



k-Selection & Median: Improved Algorithm(Cont'd)

- Use median of medians as pivot



- $T(n) < O(n) + T(n/5) + T(3n/4)$

Improved Selection

```
ImprovedSelect(A, p, r, i)
    if (p = r) then
        return A[p]
    else N = r – p + 1
    Partition A[p..r] into subsets of 5 elements and collect
    all the medians of the subsets in B[1..(N/5)].
    Pivot = ImprovedSelect (B, 1, ⌈N/5⌉, ⌈N/10⌉ )
    q = PivotPartition (A, p, r, Pivot)
    k = q – p + 1
    if (i = k)
        return A[i]
    else if (i < k)
        return RandomizedSelect(A, p, q-1, i)
    else
        return RandomizedSelect(A, q+1, r, i-k)
```