Sorting Algorithms

- Number of Comparisons
- Number of Data Movements
- Additional Space Requirements

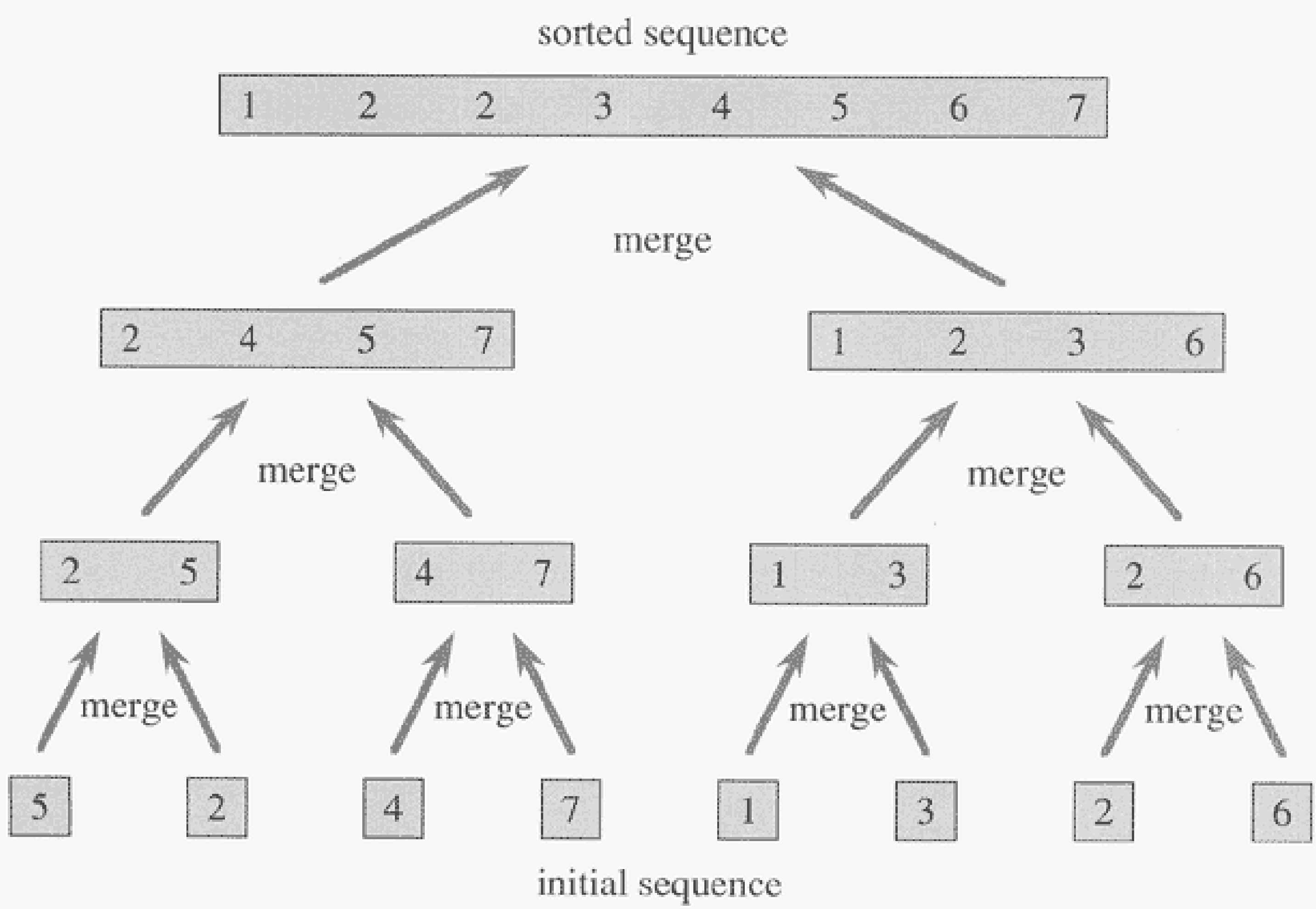
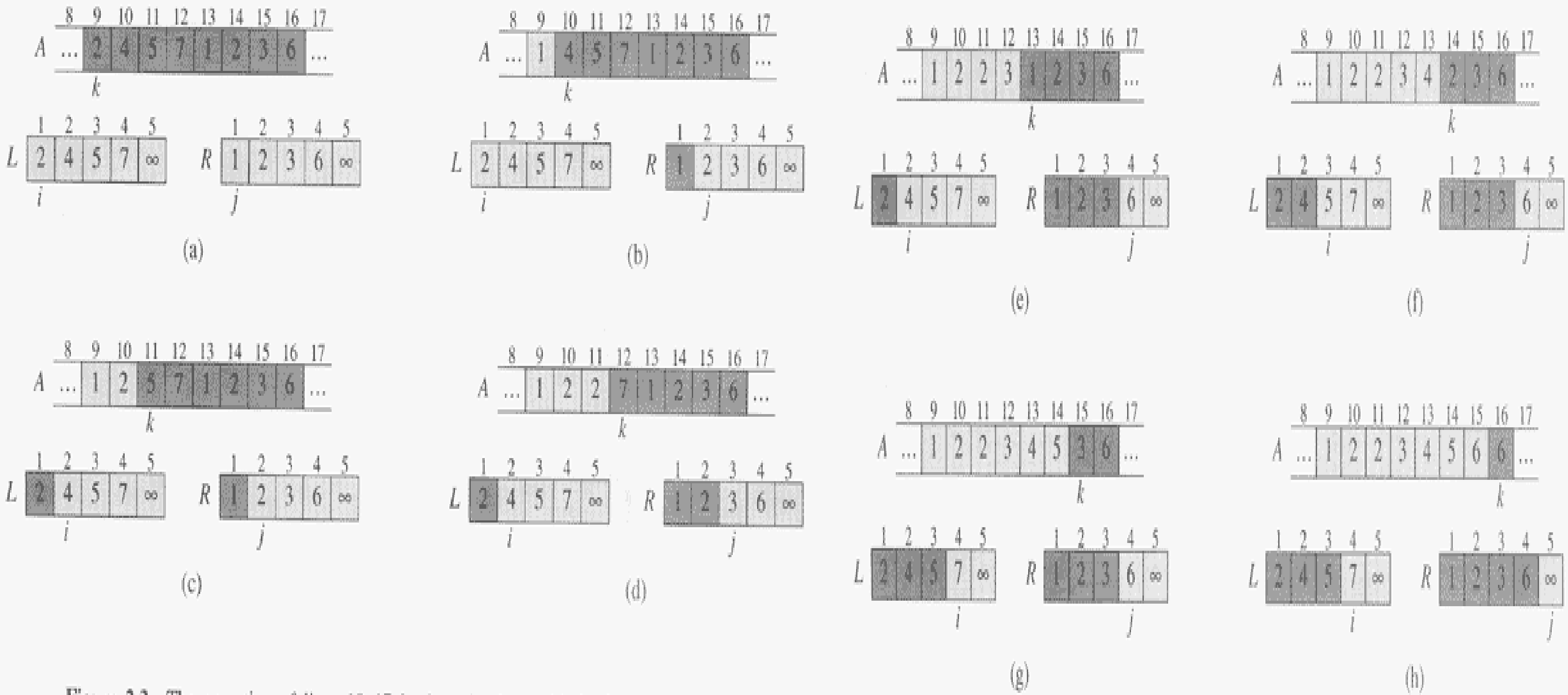


Figure 2.4 The operation of merge sort on the array $A = \langle 5, 2, 4, 7, 1, 3, 2, 6 \rangle$. The lengths of the sorted sequences being merged increase as the algorithm progresses from bottom to top.



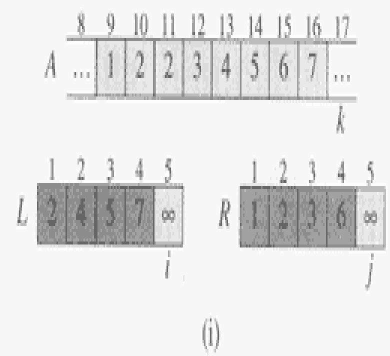


Figure 2.3 The operation of lines 10–17 in the call MERGE$(A, 9, 12, 16)$, when the subarray $A[9..16]$ contains the sequence $\langle 2, 4, 5, 7, 1, 2, 3, 6 \rangle$. After copying and inserting sentinels, the array L contains $\langle 2, 4, 5, 7, \infty \rangle$, and the array R contains $\langle 1, 2, 3, 6, \infty \rangle$. Lightly shaded positions in A contain their final values, and lightly shaded positions in L and R contain values that have yet to be copied back into A. Taken together, the lightly shaded positions always comprise the values originally in $A[9..16]$, along with the two sentinels. Heavily shaded positions in A contain values that will be copied over, and heavily shaded positions in L and R contain values that have already been copied back into A. **(a)–(h)** The arrays A, L, and R, and their respective indices k, i, and j prior to each iteration of the loop of lines 12–17. **(i)** The arrays and indices at termination. At this point, the subarray in $A[9..16]$ is sorted, and the two sentinels in L and R are the only two elements in these arrays that have not been copied into A.

```
MERGE(A, p, q, r)
n1 ← q − p + 1
n2 ← r − q
create arrays L[1 .. n1 + 1] and R[1 .. n2 + 1]
for i ← 1 to n1
     do L[i] ← A[p + i − 1]
for j ← 1 to n2
     do R[j] ← A[q + j]
L[n1 + 1] ← ∞
R[n2 + 1] ← ∞
i ← 1
j ← 1
for k ← p to r
     do if L[i] ≤ R[j]
           then A[k] ← L[i]
                i ← i + 1
           else A[k] ← R[j]
                j ← j + 1
```

Assumption: Array A is sorted from positions p to q and also from positions q+1 to r.

O(n)

MERGE-SORT(A, p, r)

if $p < r$

  then $q \leftarrow \lfloor (p + r)/2 \rfloor$

    MERGE-SORT(A, p, q)

    MERGE-SORT$(A, q + 1, r)$

    MERGE(A, p, q, r)

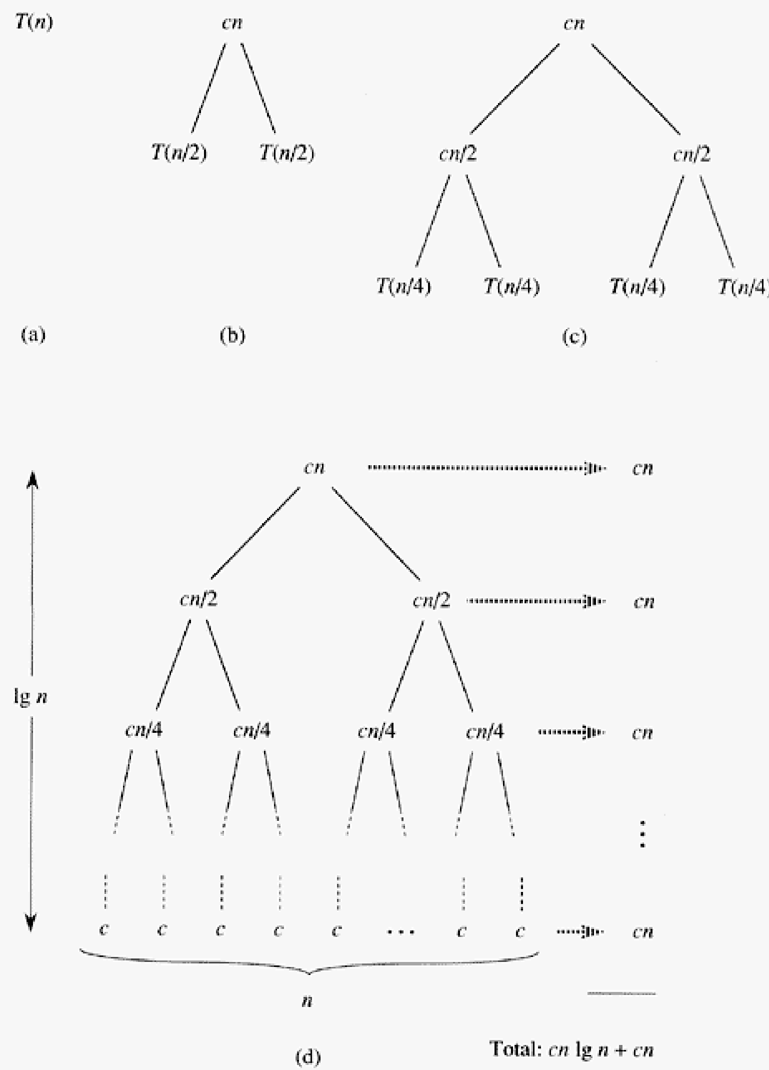
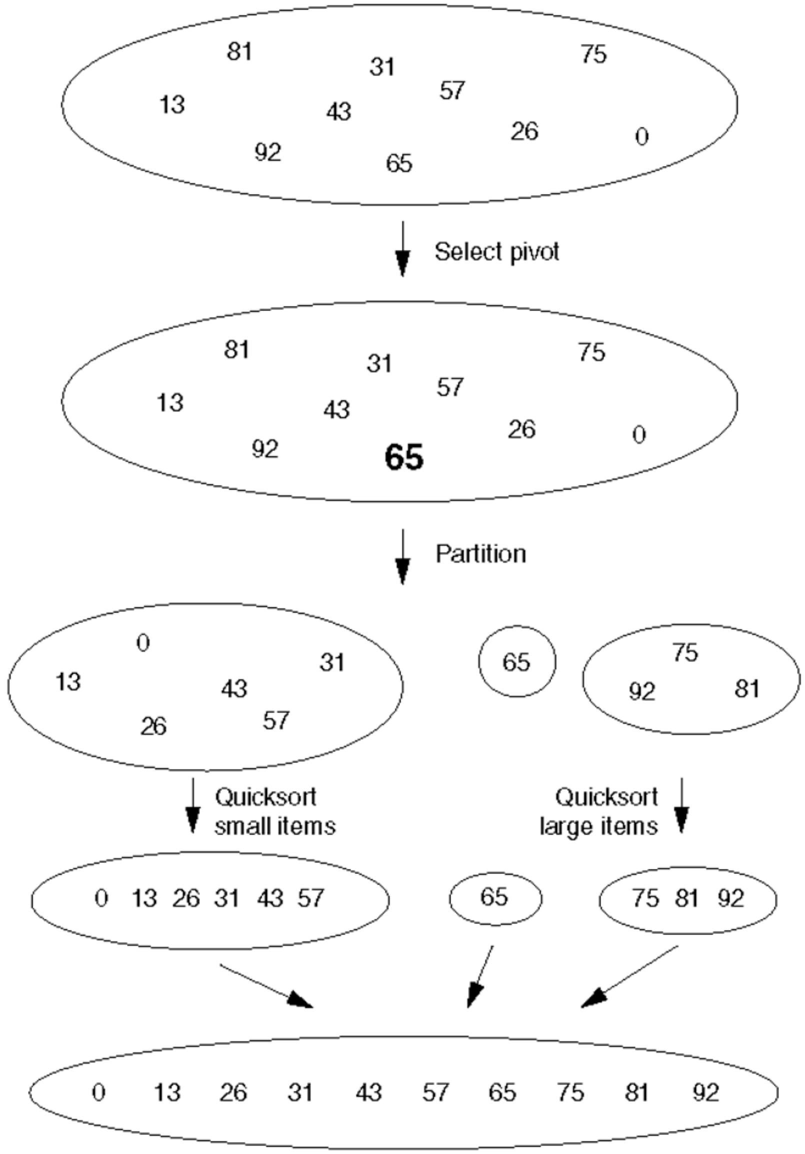


Figure 2.5 The construction of a recursion tree for the recurrence $T(n) = 2T(n/2) + cn$. Part **(a)** shows $T(n)$, which is progressively expanded in **(b)–(d)** to form the recursion tree. The fully expanded tree in part (d) has $\lg n + 1$ levels (i.e., it has height $\lg n$, as indicated), and each level contributes a total cost of cn. The total cost, therefore, is $cn \lg n + cn$, which is $\Theta(n \lg n)$.

Figure 8.10 Quicksort



Last step of Partition

Figure A If 6 is used as pivot, the end result after partitioning is as shown in the Figure B.

2	1	4	5	0	3	9	8	7	6

Figure B Result after Partitioning

2	1	4	5	0	3	6	8	7	9

QuickSort

$\text{QUICKSORT}(array\ A, int\ p, int\ r)$

if $(p < r)$
    then $q \leftarrow \text{PARTITION}(A, p, r)$
        $\text{QUICKSORT}(A, p, q - 1)$
        $\text{QUICKSORT}(A, q + 1, r)$

To sort array call $\text{QUICKSORT}(A, 1, length[A])$.

$\text{PARTITION}(array\ A, int\ p, int\ r)$

$x \leftarrow A[r]$ $\triangleright$ Choose **pivot**
$i \leftarrow p - 1$
for $j \leftarrow p$ **to** $r - 1$
    do if $(A[j] \leq x)$
        then $i \leftarrow i + 1$
            exchange $A[i] \leftrightarrow A[j]$
exchange $A[i + 1] \leftrightarrow A[r]$
return $i + 1$

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Problems with QuickSort

- Bad pivot choices can lead to worst-case behavior of $O(n^2)$
- High cost of recursive calls
- Unstable sorting algorithm
- Lots of potential data movements.

Suggestions for improvement

- Improved choice of pivot to improve worst-case time complexity
  - Randomly choose pivot
  - Pick median of three choices (deterministic or random)
- Huge cost incurred by recursive calls especially when the sublists get smaller and smaller
  - When the sublist becomes smaller than some threshold, call InsertionSort or some other non-recursive sorting algorithm

Questions to ask

- Worst-case versus average-case behavior of the sorting algorithms
- Why is QuickSort fast in practice as opposed to others?
- How can we improve on it?
- How is it different from the divide-and-conquer algorithm, MergeSort?

Sorting Algorithms

- SelectionSort
- InsertionSort
- BubbleSort
- ShakerSort
- QuickSort
- MergeSort
- HeapSort
- Bucket & Radix Sort
- Counting Sort

Problems to think about!

- What is the least number of comparisons you need to sort a list of 3 elements? 4 elements? 5 elements?
- How to arrange a tennis tournament in order to find the tournament champion with the least number of matches? How many tennis matches are needed?