COT 5993: Introduction to Algorithms

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Evaluation

- Exams
- Homework Assignments
- Semester Project
- Class Participation

Search

- You are asked to guess a number X that is known to be an integer lying between integers A and B. How many guesses do you need in the worst case?
  - Use binary search; Number of guesses = $\log_2(B-A)$
- You are asked to guess a positive integer X. How many guesses do you need in the worst case?
  - NOTE: No upper bound is known for the number.
  - Algorithm:
    - figure out B (by using Doubling Search)
    - perform binary search in the range B/2 through B.
  - Number of guesses = $\log_2 B + \log_2(B - B/2)$
  - Since X is between B/2 and B. So $\log_2(B/2) < \log_2 X$, we have
  - Number of guesses < $2\log_2 X - 1$

Polynomials

- Given a polynomial
  - $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_{n-1} x^{n-1} + a_n x^n$

  compute the value of the polynomial for a given value of x.
- How many additions and multiplications are needed?
  - Simple solution:
    - Number of additions = n
    - Number of multiplications = $1 + 2 + \ldots + n = n(n+1)/2$
  - Improved solution using **Horner's rule**:
    - $p(x) = a_0 + x(a_1 + x(a_2 + \ldots x(a_{n-1} + x\, a_n))\ldots))$
    - Number of additions = n
    - Number of multiplications = n

Celebrity Problem

- A **Celebrity** is one that knows nobody and that everybody knows.

Celebrity Problem:

INPUT: n persons with a $n \times n$ information matrix.

OUTPUT: Find the "celebrity", if one exists.

MODEL: Only allowable questions are:

  - *Does person* i *know person* j*?*

- Naive Algorithm: $O(n^2)$ Questions.
- Using Divide-and-Conquer: $O(n \log_2 n)$ Questions.
- Improved solution?

Celebrity Problem (Cont'd)

- **Induction Hypothesis 2:** We know how to find $n-2$ non-celebrities among a set of $n-1$ people, i.e., we know how to find at most one person among a set of $n-1$ people that could potentially be a celebrity.

- Resulting algorithm needs $[3(n-1)-1]$ questions.