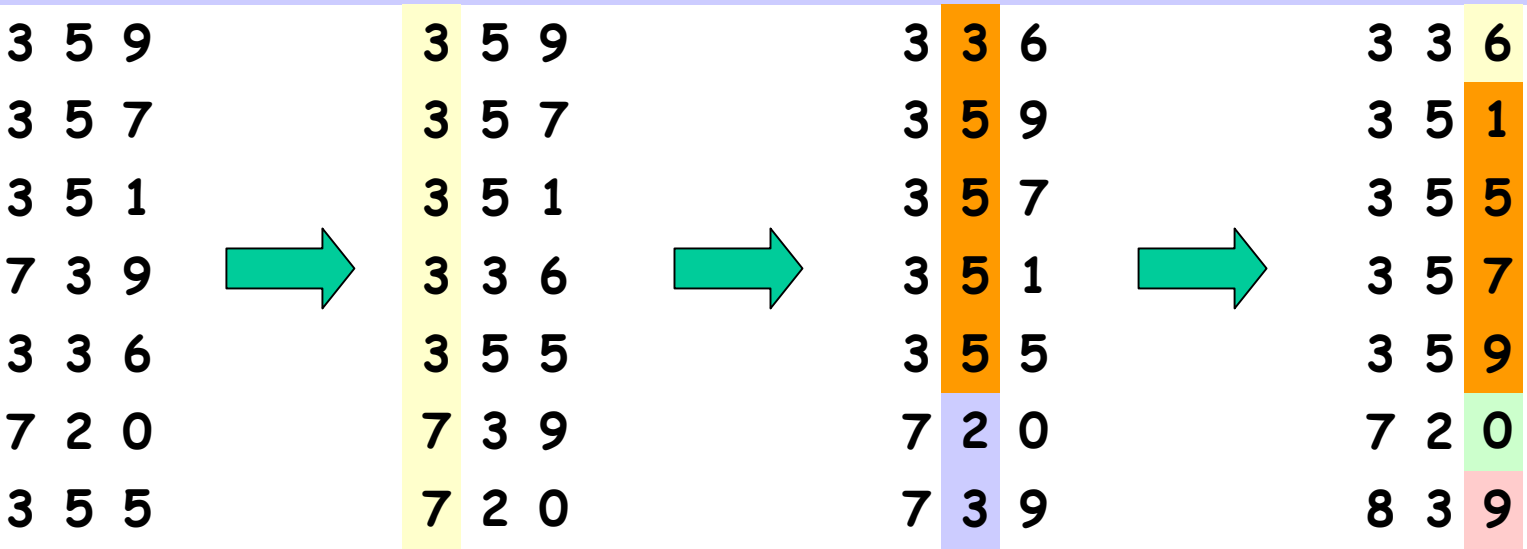
Radix Sort



Algorithm

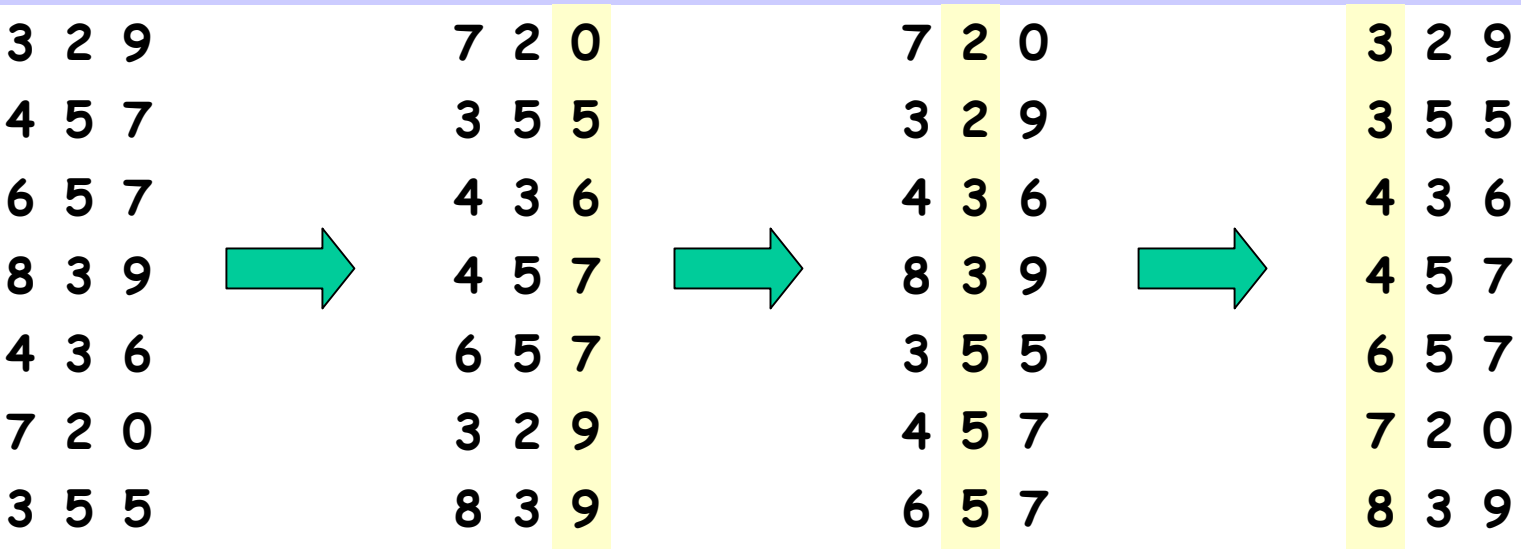
for i = 1 **to** d **do**

sort array A on digit i using any sorting algorithm

Time Complexity: $O((N+m) + (N+m^2) + \ldots + (N+m^d))$

Space Complexity: $O(m^d)$

Radix Sort



Algorithm

for i = 1 **to** d **do**

sort array A on digit i using a stable sort algorithm

Time Complexity: O((n+m)d)

Counting Sort

Initial Array

1	2	3	4	5	6	7	8
2	5	3	0	2	3	0	3

Counts

0	1	2	3	4	5
2	0	2	3	0	1

Cumulative Counts

0	1	2	3	4	5
2	2	4	7	7	8

Order Statistics

- Maximum, Minimum n-1 comparisons

7	3	1	9	4	8	2	5	0	6

- MinMax
  - 2(n-1) comparisons
  - 3n/2 comparisons
- Max and 2ndMax
  - (n-1) + (n-2) comparisons
  - ???

k-Selection; Median

- Select the k-th smallest item in list
- Naïve Solution
  - Sort;
  - pick the k-th smallest item in sorted list.

    $O(n \log n)$ time complexity
- Randomized solution: Average case $O(n)$
- Improved Solution: worst case $O(n)$

```
QuickSort(A, p, r)
    if (p < r) then
        q = Partition(A, p, r)
        QuickSort(A, p, q)
        QuickSort(A, q+1, r)

Partition(A, p, r)
  x = A[r]
  i = p-1
  for j = p to r-1 do
      if (A[j] <= x) then
          i++
          SWAP(A[i], A[j])
  SWAP(A[i+1], A[r])
  return i+1
```

Partition Procedure Revisited

- The Partition code can be rewritten so that it accepts another parameter, namely, the pivot value. Let's call this new variation as PivotPartition.
- This change does not affect its time complexity.
- RandomizedPartition as used in RandomizedSelect picks the pivot uniformly at random from among the elements in the list to be partitioned.

Randomized Selection

```
RandomizedSelect(A, p, r, i)
    if (p = r) then
        return A[p]
    q = RandomizedPartition(A, p, r)
    k = q – p + 1
    if (i = k)
        return A[i]
   else if (i < k)
        return RandomizedSelect(A, p, q-1, i)
    else
        return RandomizedSelect(A, q+1, r, i-k)
```

Randomized Selection: Rewritten

```
RandomizedSelect(A, p, r, i)
    if (p = r) then
        return A[p]
    Pivot = A[random(p,r)]
    q = PivotPartition(A, p, r, Pivot)
    k = q – p + 1
    if (i = k)
        return A[i]
    else if (i < k)
        return RandomizedSelect(A, p, q-1, i)
    else
        return RandomizedSelect(A, q+1, r, i-k)
```