Problems

20 [Problem 14-1] Suppose that we wish to keep track of a point of maximum overlap in a set of intervals – a point that has the largest number of intervals in the database overlapping it.

(a) Show that there will always be a point of maximum overlap which is an endpoint of one of the segments.

(b) Design a data structure that efficiently supports the operations INTERVAL-INSERT, INTERVAL-DELETE, and FIND-POM, which returns a point of maximum overlap.
   (Hint: Keep a red-black tree of all endpoints. Associate a value of +1 with each left endpoint, and a value of −1 for each right endpoint. Augment each node of the tree with some extra information to maintain the point of maximum overlap.)

21 Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions are ⟨5, 10, 3, 12, 5, 50, 6⟩. [Problem 15.2-1]

22 Determine an LCS of ⟨1, 0, 0, 1, 0, 1, 0⟩ and ⟨0, 1, 0, 1, 1, 0, 1⟩. [Problem 15.4-1]

23 [Problem 15.5-2] Determine the cost and structure of an optimal binary search tree for a set of n = 7 keys with the following probabilities:

<table>
<thead>
<tr>
<th>i</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>p_i</td>
<td>0.04</td>
<td>0.06</td>
<td>0.08</td>
<td>0.02</td>
<td>0.10</td>
<td>0.12</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>q_i</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

24 Professor Apollo suggests that a faster algorithm to solve the optimal triangulation problem might exist for the special case in which the weight of a triangle is its area. Is the professor’s intuition accurate? (Hint: very easy.)

25 [Problem 16.3-2] What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

   a : 1  b : 1  c : 2  d : 3  e : 5  f : 8  g : 13  h : 21

26 Solve any one of the following problems from your text: (a) Problem 15-1 (Bitonic euclidean traveling-salesperson problem), (b) Problem 15-3 (Edit distance), or (c) Problem 15-5 (Viterbi algorithm). [Note: For those of you with old editions of the text, you may consider looking at the new edition since it has more details on these problems.]