# COT 6405: Analysis of Algorithms

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# Evolution of Data Structures

- Complex problems require complex data structures.
- Simple data types → Lists.
- Applications of lists include: students roster, list of voters, grocery list, list of transactions, etc.
- Array implementation of list: random access.
- Need for list “operations” arose – “Static” vs. “dynamic” lists. “Storing” items in list vs. “Maintaining” items in list.
- Lot of research on “Sorting” and “Searching”.
- “Inserting” in a specified location in a list caused the following evolution: Array implementation → Linked list implementation.
- Other linear structures e.g., stacks, queues, etc.

# Evolution of Data Structures

- Trees made hierarchical organization of data easy to handle. Applications of trees: administrative hierarchy in a business set up, storing an arithmetic expression, organization of the functions calls of a recursive program, etc.
- Search trees (e.g., BST) were designed to make search and retrieval efficient in trees. A BST may not allow fast search or retrieval, if it is very unbalanced, since the time complexities of the operations depended on the height of the tree.
- Graphs generalize trees; model more general networks.
- Abstract data types. Advantages include: Encapsulation of data and operations, hiding of unnecessary details, localization and debugging of errors, ease of use since interface is clearly specified, ease of program development, etc.

# Solving Recurrence Relations

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| Recurrence; Cond | Solution |
|---|---|
| $T(n) = T(n-1) + O(1)$ | $T(n) = O(n)$ |
| $T(n) = T(n-1) + O(n)$ | $T(n) = O(n^2)$ |
| $T(n) = T(n-c) + O(1)$ | $T(n) = O(n)$ |
| $T(n) = T(n-c) + O(n)$ | $T(n) = O(n^2)$ |
| $T(n) = 2T(n/2) + O(n)$ | $T(n) = O(n \log n)$ |
| $T(n) = aT(n/b) + O(n);$<br>$a = b$ | $T(n) = O(n \log n)$ |
| $T(n) = aT(n/b) + O(n);$<br>$a < b$ | $T(n) = O(n)$ |
| $T(n) = aT(n/b) + f(n);$<br>$f(n) = O(n^{\log_b a - \epsilon})$ | $T(n) = O(n)$ |
| $T(n) = aT(n/b) + f(n);$<br>$f(n) = O(n^{\log_b a})$ | $T(n) = \Theta(n^{\log_b a} \log n)$ |
| $T(n) = aT(n/b) + f(n);$<br>$f(n) = \Theta(f(n))$<br>$af(n/b) \leq cf(n)$ | $T(n) = \Omega(n^{\log_b a} \log n)$ |

# Sorting Algorithms

- Selection Sort
- Insertion Sort
- Bubble Sort
- Shaker Sort
- Shell Sort
- Merge Sort
- Heap Sort
- Quick Sort

- Bucket & Radix Sort
- Counting Sort

# Algorithm Invariants

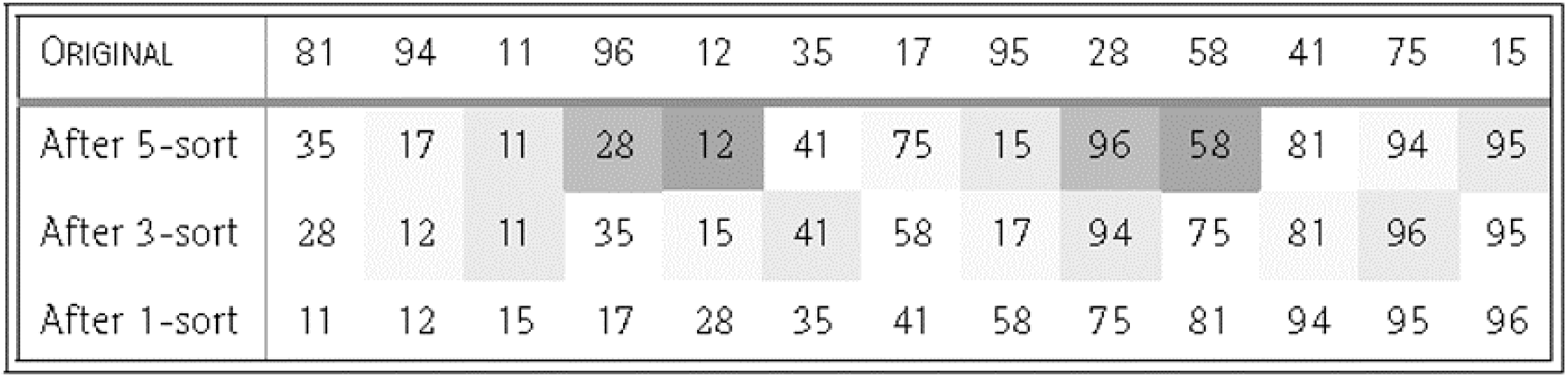
- Selection Sort
  - iteration k: the k smallest items are in correct location.
- Insertion Sort
  - iteration k: the first k items are in sorted order.
- Bubble Sort
  - In each pass, every item that does not have a smaller item after it, is moved as far up in the list as possible.
  - Iteration k: k smallest items are in the correct location.
- Shaker Sort
  - In each odd (even) numbered pass, every item that does not have a smaller (larger) item after it, is moved as far up (down) in the list as possible.
  - Iteration k: the k/2 smallest and largest items are in the correct location.

# Algorithm Invariants (Cont’d)

- Merge (many lists)
  - Iteration k: the k smallest items from the lists are merged.
- Heapify
  - Iteration with i = k: Subtrees with roots at indices k or larger satisfy the heap property.
- HeapSort
  - Iteration k: Largest k items are in the right location.
- Partition (two sublists)
  - Iteration k (with pointers at i and j): items in locations [1..I] (locations [i+1..j]) are at least as small (large) as the pivot.

# Figure 8.5

Shellsort after each pass if the increment sequence is {1, 3, 5}



| ORIGINAL | 81 | 94 | 11 | 96 | 12 | 35 | 17 | 95 | 28 | 58 | 41 | 75 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| After 5-sort | 35 | 17 | 11 | 28 | 12 | 41 | 75 | 15 | 96 | 58 | 81 | 94 | 95 |
| After 3-sort | 28 | 12 | 11 | 35 | 15 | 41 | 58 | 17 | 94 | 75 | 81 | 96 | 95 |
| After 1-sort | 11 | 12 | 15 | 17 | 28 | 35 | 41 | 58 | 75 | 81 | 94 | 95 | 96 |

# ShellSort

```
public static void shellsort( Comparable [ ] a )
  {
      for( int gap = a.length / 2; gap > 0;
                 gap = gap == 2 ? 1 : (int) ( gap / 2.2 ) )
          for( int i = gap; i < a.length; i++ )
          {
              Comparable tmp = a[ i ];
              int j = i;

              for( ; j >= gap && tmp.compareTo( a[ j - gap ] ) < 0; j -= gap )
                  a[ j ] = a[ j - gap ];
              a[ j ] = tmp;
          }
  }
```

# Sorting Algorithms

- Number of Comparisons
- Number of Data Movements
- Additional Space Requirements

# Sorting Algorithms

- Selection Sort
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- Counting Sort

# Animation Demos

http://www-cse.uta.edu/~holder/courses/cse2320/lectures/applets/sort1/heapsort.html

http://cg.scs.carleton.ca/~morin/misc/sortalg/

```
QuickSort(A, p, r)
    if (p < r) then
        q = Partition(A, p, r)
        QuickSort(A, p, q-1)
        QuickSort(A, q+1, r)

Partition(A, p, r)
  x = A[r]
  i = p-1
  for j = p to r-1 do
      if A[j] <= x) then
          i++
          exchange(A[i], A[j])
exchange(A[i+1], A[r])
return i+1
```

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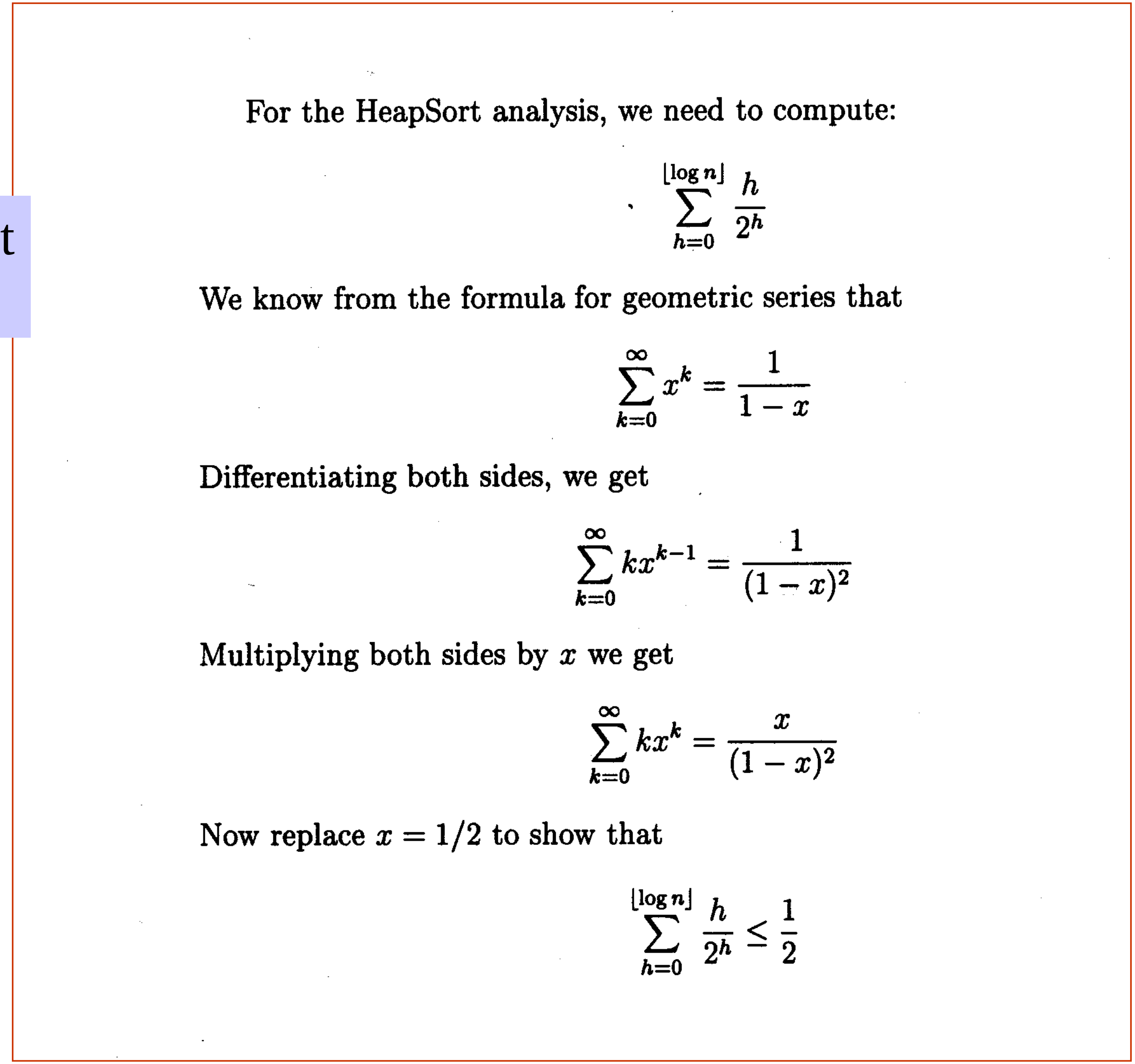
# HeapSort Analysis



For the HeapSort analysis, we need to compute:

$$\sum_{h=0}^{\lfloor \log n \rfloor} \frac{h}{2^h}$$

We know from the formula for geometric series that

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

Differentiating both sides, we get

$$\sum_{k=0}^{\infty} k x^{k-1} = \frac{1}{(1-x)^2}$$

Multiplying both sides by $x$ we get

$$\sum_{k=0}^{\infty} k x^k = \frac{x}{(1-x)^2}$$

Now replace $x = 1/2$ to show that

$$\sum_{h=0}^{\lfloor \log n \rfloor} \frac{h}{2^h} \leq \frac{1}{2}$$

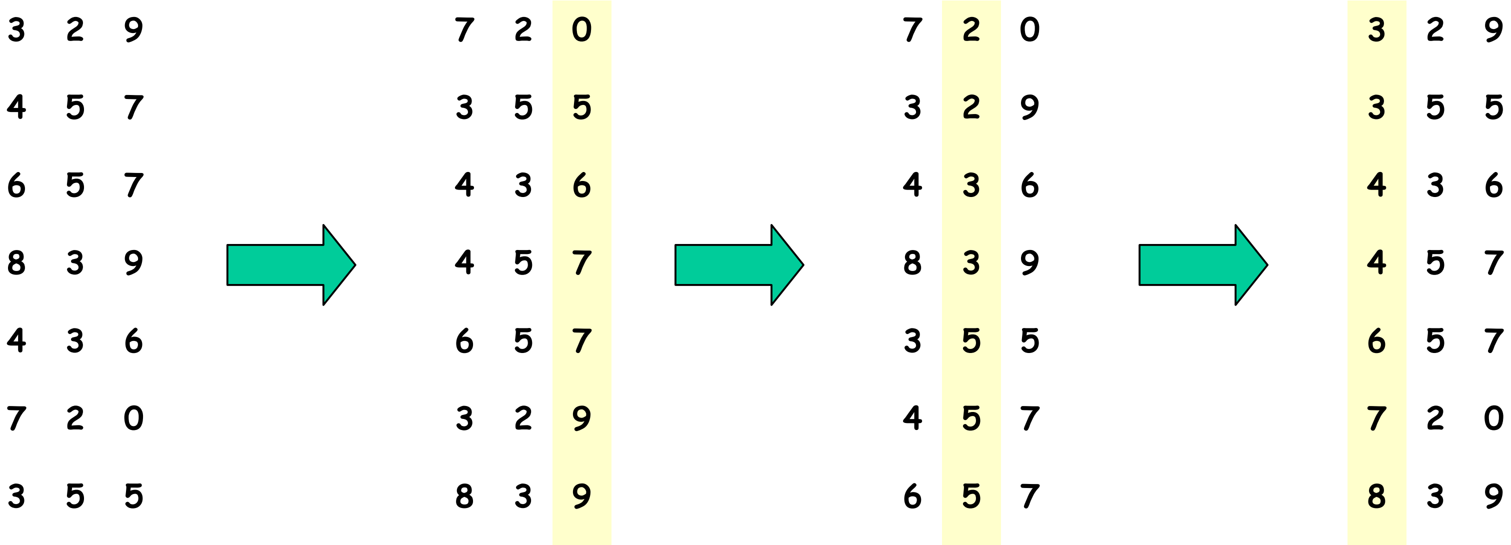
# Bucket Sort

- N values in the range [a..a+m-1]
- For e.g., sort a list of 50 scores in the range [0..9].
- Algorithm
  - Make m buckets [a..a+m-1]
  - As you read elements throw into appropriate bucket
  - Output contents of buckets [0..m] in that order
- Time O(N+m)

# Stable Sort

- A sort is stable if equal elements appear in the same order in both the input and the output.
- Which sorts are stable? Homework!

# Radix Sort



| | | | |
|---|---|---|---|
| 3 2 9 | 7 2 0 | 7 2 0 | 3 2 9 |
| 4 5 7 | 3 5 5 | 3 2 9 | 3 5 5 |
| 6 5 7 | 4 3 6 | 4 3 6 | 4 3 6 |
| 8 3 9 | 4 5 7 | 8 3 9 | 4 5 7 |
| 4 3 6 | 6 5 7 | 3 5 5 | 6 5 7 |
| 7 2 0 | 3 2 9 | 4 5 7 | 7 2 0 |
| 3 5 5 | 8 3 9 | 6 5 7 | 8 3 9 |

**Algorithm**

```
for i = 1 to d do
        sort array A on digit i using a stable sort algorithm
```

Time Complexity: $O((n+k)d)$

# Counting Sort

Initial Array

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 2 | 5 | 3 | 0 | 2 | 3 | 0 | 3 |

Counts

| 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 2 | 0 | 2 | 3 | 0 | 1 |

Cumulative Counts

| 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 2 | 2 | 4 | 7 | 7 | 8 |