COT 6936: Topics in Algorithms

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http://www.cs.fiu.edu/~giri/teach/COT6936_S10.html

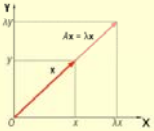
https://online.cis.fiu.edu/portal/course/view.php?id=427

Spectral Methods

- Graph Connectivity problems
  - Google Page Rank
- Graph Partitioning problems
  - Clustering (even linearly non-separable case)
- Markov Chain Mixing problems
  - Random walks in graphs

Matrices and Eigenvalues

- Array of values
- Linear Transformation



- Eigenvalues and Eigenvectors
  - $Ax = \lambda x$
  - Under transformation A, eigenvectors only experience change in magnitude, not direction
  - $A = Q \Lambda Q^{-1}$

Graph Bisection

- Construct adjacency matrix A
- Construct Laplacian L = D – A
- L is positive semi-definite (PSD); has non-neg eigenvalues; has smallest eigenvalue = 0
- Second eigenvector provides information about bisection.
  - Signs of 2nd eigenvector give a good bisection
  - Extreme case: Connected components have constant values in 2nd eigenvector

Graph Bisection (Continued)

- Eigenvalues indicate strength of bisection
- How to get bisections with n/2 vertices?
  - Use median value in second eigenvector
- How to get k partitions?
  - Perform bisections recursively
  - Use more eigenvectors

Spectral Clustering: Strategy

- Given data points and a distance function, construct a weighted graph
- Let A be its adjacency matrix; let D be diagonal matrix with degrees along diagonal
- Construct Laplacian L (PSD, non-neg eigenv.)
  - Unnormalized: $L = D - A$
  - Normalized symmetric: $L = D^{-1/2}LD^{1/2}$
  - Random Walk: $L = D^{-1}L$
- Matrix L_k has cols = first k eigenvectors of L
- Cluster rows of L_k.

Spectral Clustering

- Need distance measure (need not be a metric), i.e., triangle inequality not needed
- Not Model-based
- Global method
- Turns discrete problem into continuous