

COT 6936: Topics in Algorithms

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http://www.cs.fiu.edu/~giri/teach/COT6936_S12.html

<https://moodle.cis.fiu.edu/v2.1/course/view.php?id=174>

COT 6936: Topics in Algorithms

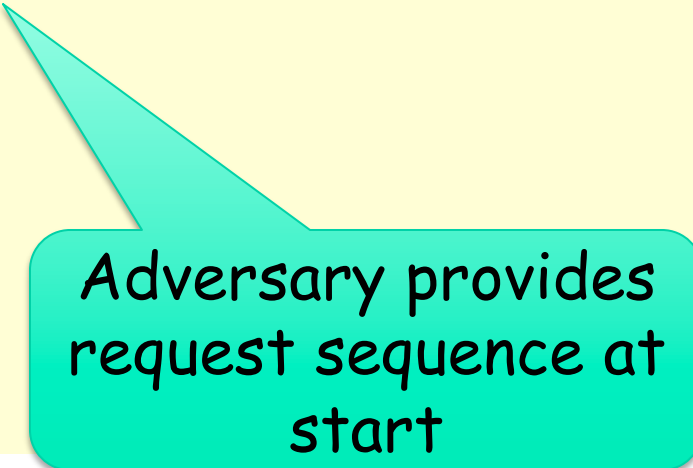
Online Algorithms

How to Analyze Online Algorithms?

- Competitive analysis
 - Compare with optimal offline algorithm (OPT)
- Algorithm A is **α -competitive** if there exists constants b such that for every sequence of inputs σ :
 - $\text{cost}_A(\sigma) \leq \alpha \text{cost}_{\text{OPT}}(\sigma) + b$

How to Analyze Rand Online Algorithms?

- Algorithm A is **α -competitive** if there exists constants b such that for every sequence of inputs σ :
 - $\text{cost}_A(\sigma) \leq \alpha \text{cost}_{\text{OPT}}(\sigma) + b$
- Randomized Algorithm R is **α -competitive** if there exists constants b such that for every sequence of inputs σ :
 - $E[\text{cost}_R(\sigma)] \leq \alpha \text{cost}_{\text{OPT}}(\sigma) + b$



Adversary provides request sequence at start

Randomized Online algorithms

- Lower bound does not apply to randomized algorithms
 - Lower bound on randomized algorithms = H_k
- Proof uses 2 main principles
 - Cover time of a random walk on K_{k+1} is kH_k
 - Lower bound on competitiveness of randomized algorithms equals competitiveness of best deterministic algorithm A on "worst-case" distribution on request sequence
- H_k is k -th Harmonic number and $< \ln(k) + 1$

Randomized Algorithm: RANDOM

- On a miss:
 - Evict an item chosen uniformly at random from all k items
- RANDOM is k -competitive

Marker Algorithm

- Each of k pages has a **marker** bit
- Algorithm proceeds in **rounds** with invariant:
 - At start of round all pages are **unmarked**
- In each round
 - If request is a hit: mark page
 - If request is a miss:
 - If all pages are marked: start next round and unmark all pages
 - Replace (arbitrary) unmarked page and mark it
- Intuition: k pages accessed each round

Marker Algorithm

- Marker algorithm is k -competitive
 - In each round, algorithm has k misses
 - OPT has at least one miss because $k+1$ distinct pages are accessed including the last access from previous round

Randomized Marker Algorithm

- Each of k pages has a **marker** bit
 - Algorithm proceeds in **rounds** with invariant:
 - At start of round all pages are **unmarked**
 - In each round
 - If request is a hit: mark page
 - If request is a miss:
 - If all cache pages are marked: start next round by unmarking all locations
 - Else evict (randomly) unmarked page and mark it
 - This algorithm is $2H_k$ -competitive
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