COP 4516: Competitive Programming and Problem Solving

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Evaluation

• Exam/Competition	50%
• Solving Problems	40%
• Attendance	5%
• Class Participation	5%

Sorting

- Input is a list of n items that can be compared.
- Output is an ordered list of those n items.
- Fundamental problem that has received a lot of attention over the years.
- Used in many applications.
- Scores of different algorithms exist.
- Task: To compare algorithms
  - On what bases?
    - Time
    - Space
    - Other

Sorting Algorithms

- Number of Comparisons
- Number of Data Movements
- Additional Space Requirements

Sorting Algorithms

- SelectionSort
- InsertionSort
- BubbleSort
- ShakerSort
- MergeSort
- HeapSort
- QuickSort
- Bucket & Radix Sort
- Counting Sort

SelectionSort

```
SELECTIONSORT(array A)
N ← length[A]
for p ← 1 to N
        do Compute j, the index of the
           smallest item in A[p..N]
           Swap A[p] and A[j]
```

SelectionSort

```
SELECTIONSORT(array A)
N ← length[A]
for p ← 1 to N
    do ▷ Compute j
        j ← p
        for m ← p + 1 to N
            do if (A[m] < A[j])
                then j ← m
        ▷ Swap A[p] and A[j]
        temp ← A[p]
        A[p] ← A[j]
        A[j] ← temp
```

SelectionSort

```
SelectionSort(array A)
N ← length[A]
for p ← 1 to N
      do ▷ Compute j
         j ← p
         for m ← p + 1 to N
               do if (A[m] < A[j])
                     then j ← m
         ▷ Swap A[p] and A[j]
         temp ← A[p]
         A[p] ← A[j]
         A[j] ← temp
```

$O(n^2)$ time

$O(1)$ space

Solving Recurrence Relations

Page 62, [CLR]

Recurrence; Cond	Solution
$T(n) = T(n-1) + O(1)$	$T(n) = O(n)$
$T(n) = T(n-1) + O(n)$	$T(n) = O(n^2)$
$T(n) = T(n-c) + O(1)$	$T(n) = O(n)$
$T(n) = T(n-c) + O(n)$	$T(n) = O(n^2)$
$T(n) = 2T(n/2) + O(n)$	$T(n) = O(n \log n)$
$T(n) = aT(n/b) + O(n)$; $a = b$	$T(n) = O(n \log n)$
$T(n) = aT(n/b) + O(n)$; $a < b$	$T(n) = O(n)$
$T(n) = aT(n/b) + f(n)$; $f(n) = O(n^{\log_b a - \epsilon})$	$T(n) = O(n)$
$T(n) = aT(n/b) + f(n)$; $f(n) = O(n^{\log_b a})$	$T(n) = \Theta(n^{\log_b a} \log n)$
$T(n) = aT(n/b) + f(n)$; $f(n) = \Theta(f(n))$ $af(n/b) \leq cf(n)$	$T(n) = \Omega(n^{\log_b a} \log n)$

```
INSERTION-SORT(A)
for j ← 2 to length[A]
     do key ← A[j]
        ▷ Insert A[j] into the sorted sequence A[1 . . j − 1].
        i ← j − 1
        while i > 0 and A[i] > key
            do A[i + 1] ← A[i]
               i ← i − 1
        A[i + 1] ← key
```

Loop invariants and the correctness of insertion sort

INSERTION-SORT(*A*)	*cost*	*times*
for $j \leftarrow 2$ **to** *length*[*A*]	c_1	n
do $key \leftarrow A[j]$	c_2	$n-1$
▷ Insert $A[j]$ into the sorted sequence $A[1 \,.\,.\, j-1]$.	0	$n-1$
$i \leftarrow j-1$	c_4	$n-1$
while $i > 0$ and $A[i] > key$	c_5	$\sum_{j=2}^{n} t_j$
do $A[i+1] \leftarrow A[i]$	c_6	$\sum_{j=2}^{n} (t_j - 1)$
$i \leftarrow i-1$	c_7	$\sum_{j=2}^{n} (t_j - 1)$
$A[i+1] \leftarrow key$	c_8	$n-1$

$O(n^2)$ time

$O(1)$ space

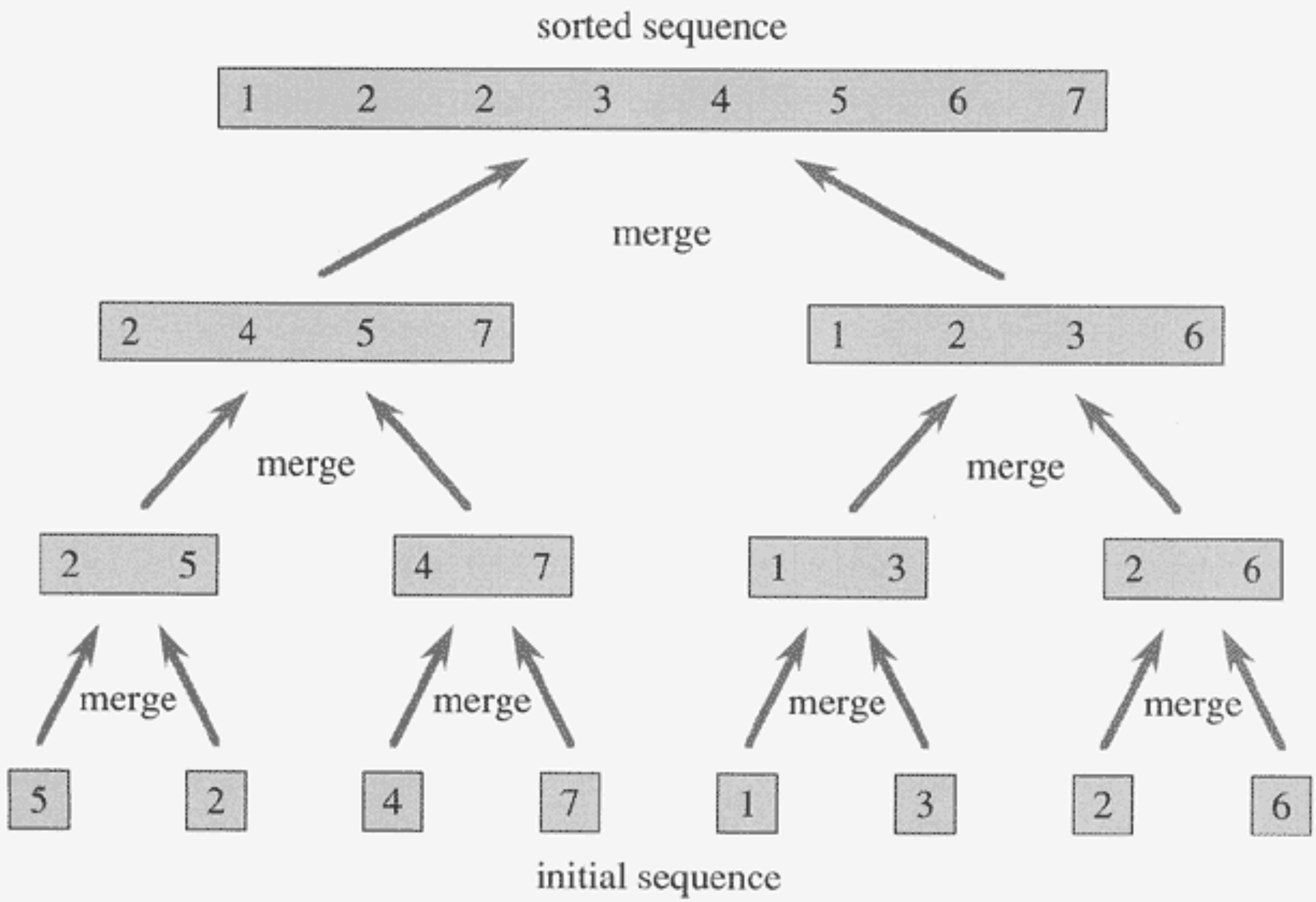
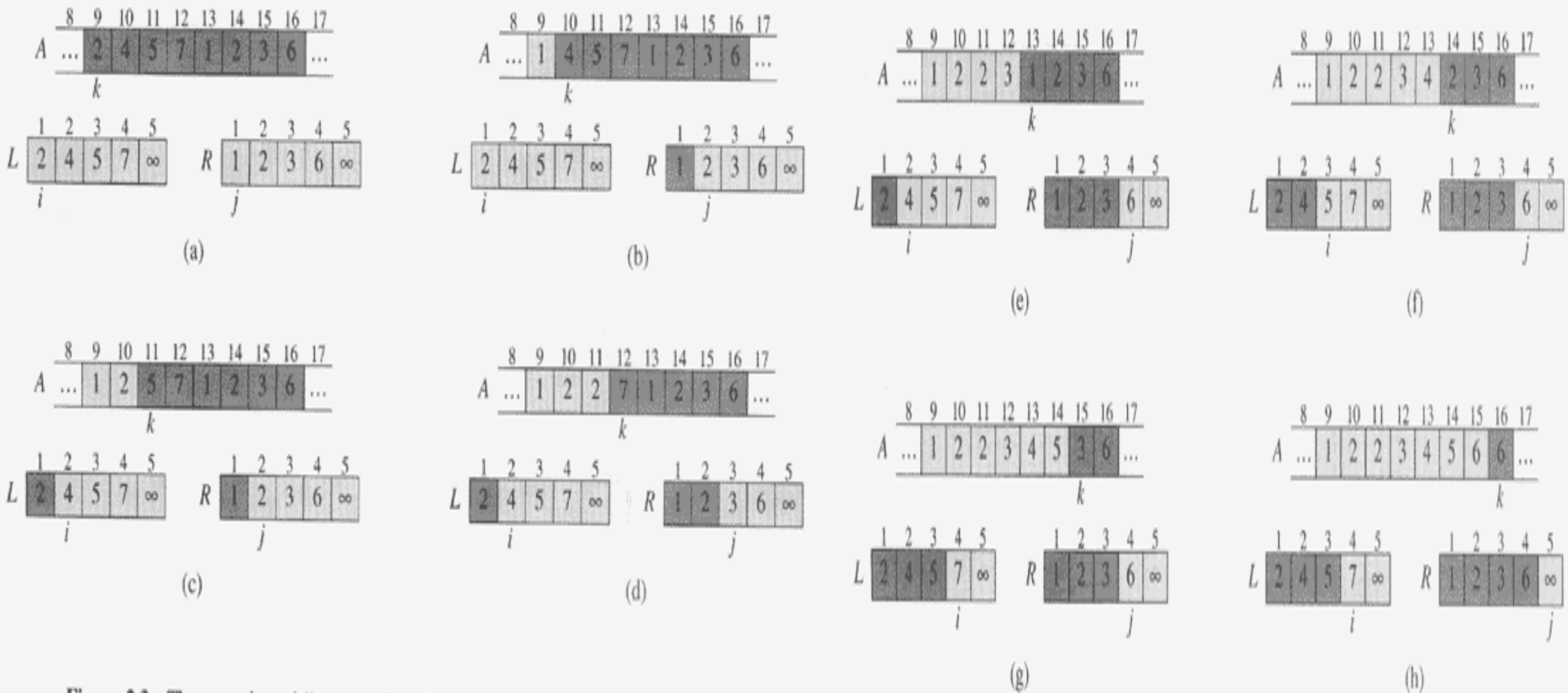


Figure 2.4 The operation of merge sort on the array $A = \langle 5, 2, 4, 7, 1, 3, 2, 6 \rangle$. The lengths of the sorted sequences being merged increase as the algorithm progresses from bottom to top.



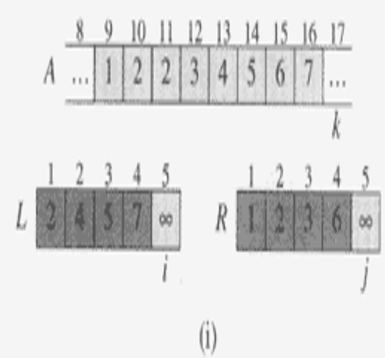


Figure 2.3 The operation of lines 10–17 in the call MERGE$(A, 9, 12, 16)$, when the subarray $A[9..16]$ contains the sequence $\langle 2, 4, 5, 7, 1, 2, 3, 6 \rangle$. After copying and inserting sentinels, the array L contains $\langle 2, 4, 5, 7, \infty \rangle$, and the array R contains $\langle 1, 2, 3, 6, \infty \rangle$. Lightly shaded positions in A contain their final values, and lightly shaded positions in L and R contain values that have yet to be copied back into A. Taken together, the lightly shaded positions always comprise the values originally in $A[9..16]$, along with the two sentinels. Heavily shaded positions in A contain values that will be copied over, and heavily shaded positions in L and R contain values that have already been copied back into A. **(a)–(h)** The arrays A, L, and R, and their respective indices k, i, and j prior to each iteration of the loop of lines 12–17. **(i)** The arrays and indices at termination. At this point, the subarray in $A[9..16]$ is sorted, and the two sentinels in L and R are the only two elements in these arrays that have not been copied into A.

```
MERGE(A, p, q, r)
n1 ← q − p + 1
n2 ← r − q
create arrays L[1 .. n1 + 1] and R[1 .. n2 + 1]
for i ← 1 to n1
      do L[i] ← A[p + i − 1]
for j ← 1 to n2
      do R[j] ← A[q + j]
L[n1 + 1] ← ∞
R[n2 + 1] ← ∞
i ← 1
j ← 1
for k ← p to r
      do if L[i] ≤ R[j]
            then A[k] ← L[i]
                 i ← i + 1
            else A[k] ← R[j]
                 j ← j + 1
```

Assumption: Array A is sorted from positions p to q and also from positions q+1 to r.

```
MERGE-SORT(A, p, r)
if p < r
   then q ← ⌊(p + r)/2⌋
        MERGE-SORT(A, p, q)
        MERGE-SORT(A, q + 1, r)
        MERGE(A, p, q, r)
```

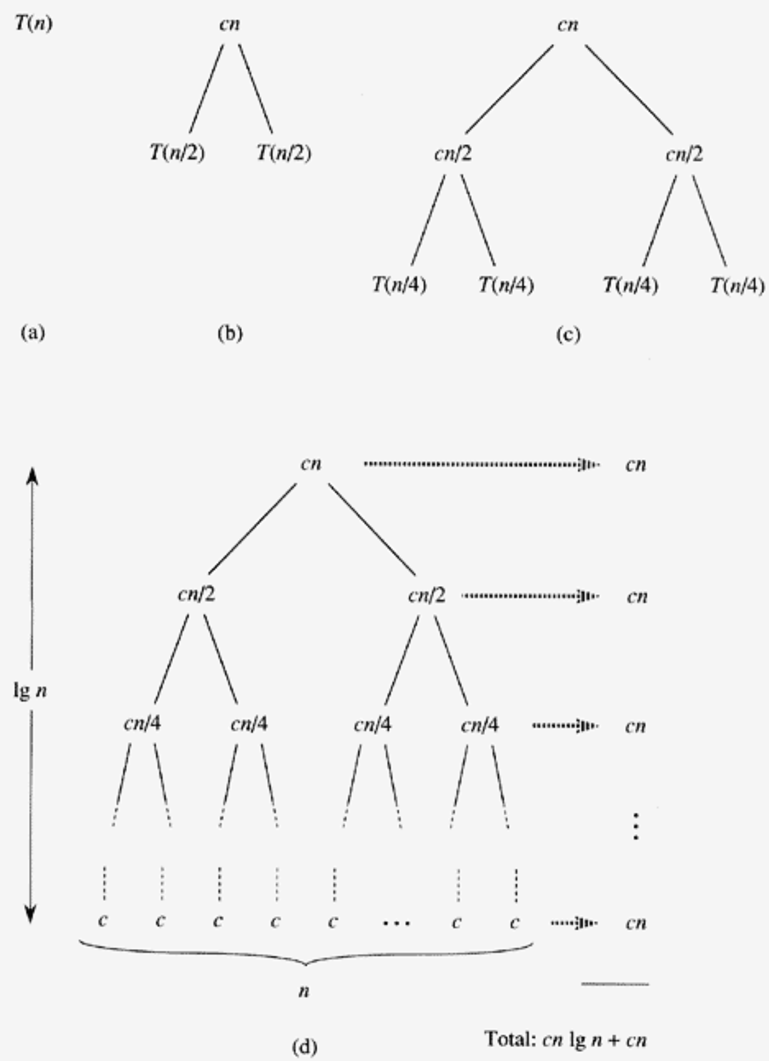


Figure 2.5 The construction of a recursion tree for the recurrence $T(n) = 2T(n/2) + cn$. Part **(a)** shows $T(n)$, which is progressively expanded in **(b)–(d)** to form the recursion tree. The fully expanded tree in part (d) has $\lg n + 1$ levels (i.e., it has height $\lg n$, as indicated), and each level contributes a total cost of cn. The total cost, therefore, is $cn \lg n + cn$, which is $\Theta(n \lg n)$.

Sorting Algorithms

- SelectionSort
- InsertionSort
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- QuickSort
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Animations

- http://cg.scs.carleton.ca/~morin/misc/sortalg/
- http://home.westman.wave.ca/~rhenry/sort/
  - time complexities on best, worst and average case
- http://vision.bc.edu/~dmartin/teaching/sorting/anim-html/quick3.html
  - runs on almost sorted, reverse, random, and unique inputs; shows code with invariants
- http://www.brian-borowski.com/Sorting/
  - comparisons, movements & stepwise animations with user data
- http://maven.smith.edu/~thiebaut/java/sort/demo.html
  - comparisons & data movements and step by step execution

Comparing O(n^2) Sorting Algorithms

- InsertionSort and SelectionSort (and ShakerSort) are roughly twice as fast as BubbleSort for small files.
- InsertionSort is the best for very small files.
- O(n^2) sorting algorithms are **NOT** useful for large random files.
- If comparisons are very expensive, then among the O(n^2) sorting algorithms, insertionsort is best.
- If data movements are very expensive, then among the O(n^2) sorting algorithms, ?? is best.

Selection

- Given a set of n items and a number k, select the k^{th} smallest item from the set.
  - $k = 1$
  - $k = n$
  - $k = n/2$
  - Arbitrary k
- General Solution:
  - Sort, then select

Problems to think about!

- What is the least number of comparisons you need to sort a list of 3 elements? 4 elements? 5 elements?
- How to arrange a tennis tournament in order to find the tournament champion with the least number of matches? How many tennis matches are needed?
- How to randomize the order of a list?

Search

- Given a set of n items, search for item x
  - Unordered list
  - Ordered list
    - Array list
    - Linked List
    - ??

Binary Search Trees