Two Stones
kattis: twostones

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Spring 2019

Game of “Two Stones”

- Given N stones arranged sequentially, Alice and Bob play a game as follows
  - In each turn they pick exactly 2 adjacent stones
  - If no more pairs of adjacent stones exist, game ends
  - **Alice wins** if
    - number of stones remaining is **odd**
  - Else **Bob wins**
- Output the winner “Alice” or “Bob” without quotes on a line

Who wins with …

- N = 1
- N = 2
- N = 3
- N = 4
- N = odd
- N = even

Challenges

- N has a large range: N <= 10 million
- There are 2 possible starts depending on who goes first
- For large N, there are many, many moves for each player at each step
- Cannot simulate every possible game to figure out who the winner would be

Method of Reduction

- For each possible scenario, what happens when one of them moves?
  - How does the situation change?
  - How does it affect who will win?
  - What is a good move? Bad move?

Invariant

- Alice wins if the number of lone stones are odd in number
- Alice wins if there are an odd number of odd sequences
- Regardless of who plays, the **parity** of the number of odd sequences remains the same
  - If it started with even, it stays even. **HOW**?
  - If it started with odd, it stays odd

Final Solution

- We only need to count the number of odd sequences in the initial set
- Since there is only one sequence of length N at start, the answer is simple
  - If N = odd, Alice wins
  - Else, Bob wins

What if …

- What if Alice’s reward is equal to the number of stones left over?

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