Convex Hull

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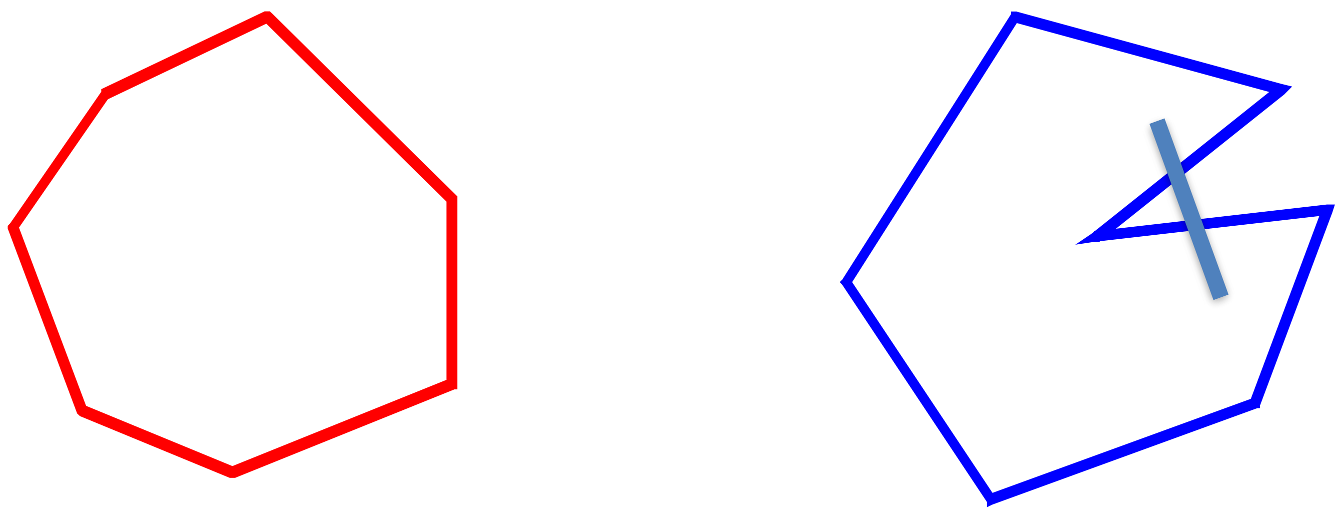
Convex Regions

- Convex region: A region in space is called convex if line joining any two points in the region is completely contained in the region.

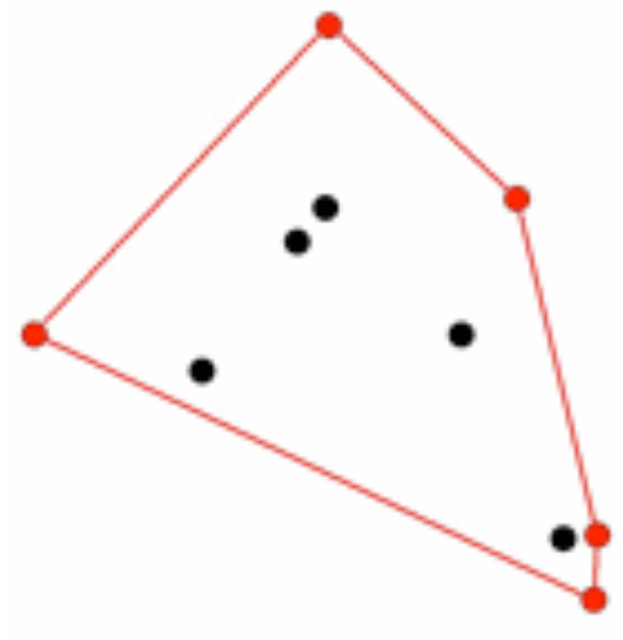
Non-convex polygons

- Convex vs Non-convex

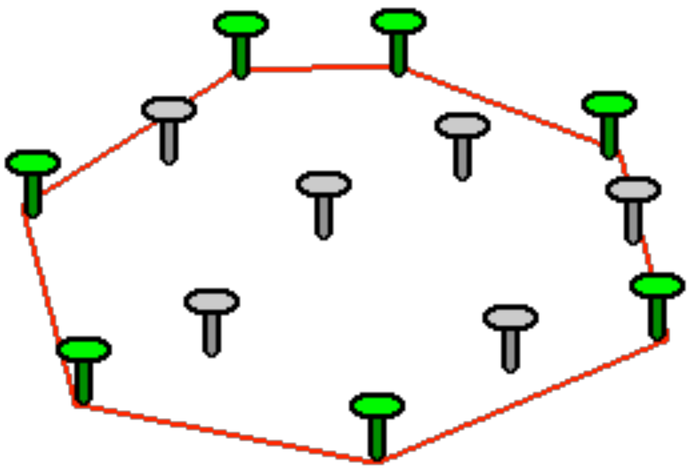


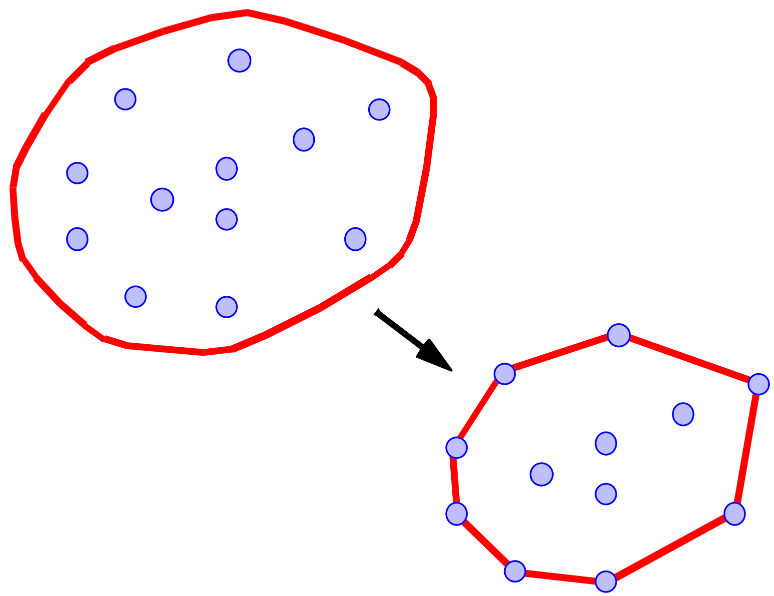
Convex Hulls and Polygons

- Convex hull of a set of points, S, is the smallest convex region containing S.

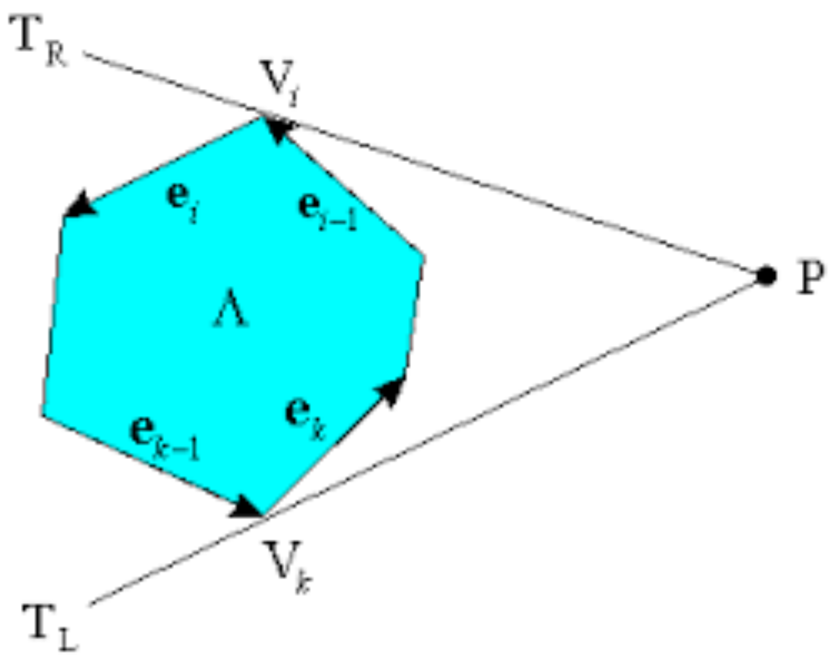


Rubber Band Analogy for Convex Hulls

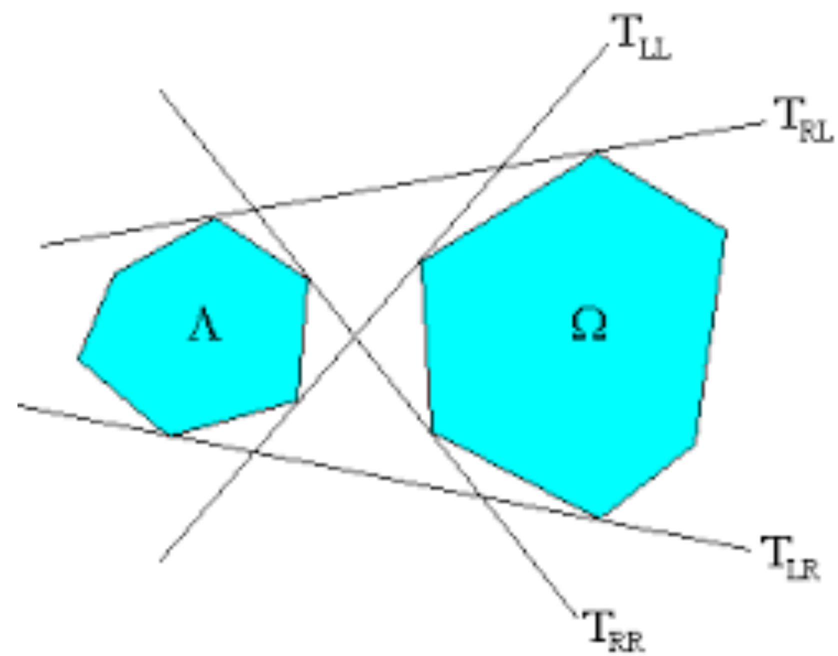




Tangents to Polygons

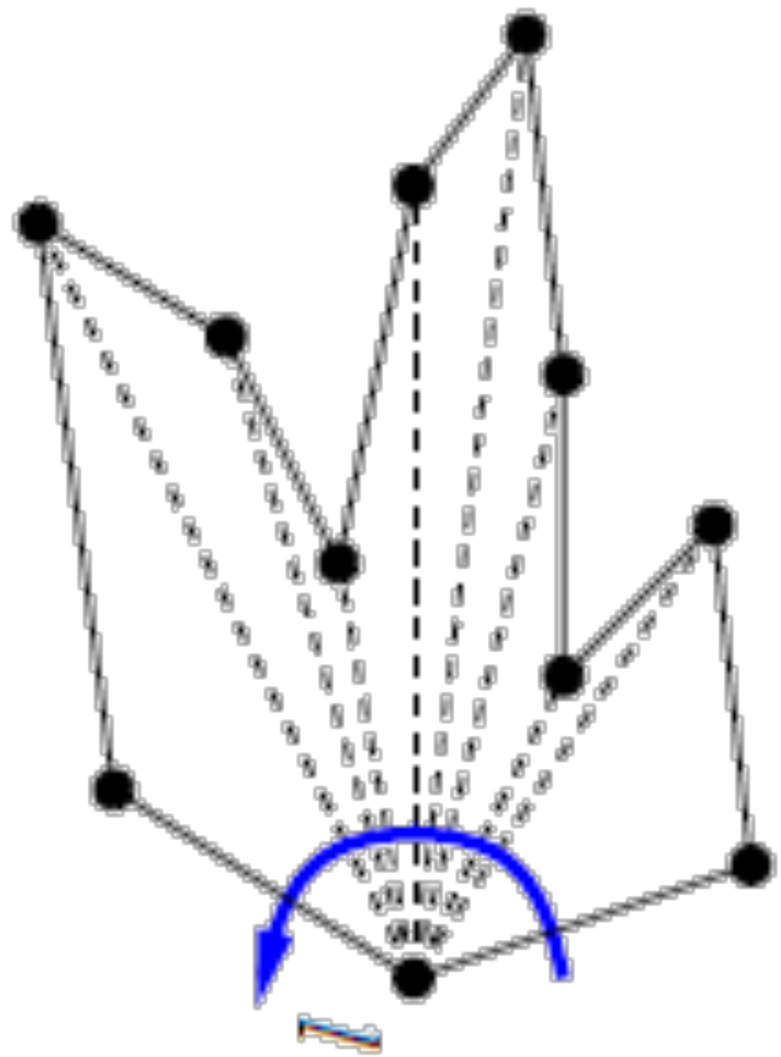


Tangents from a point



Tangents from a polygon

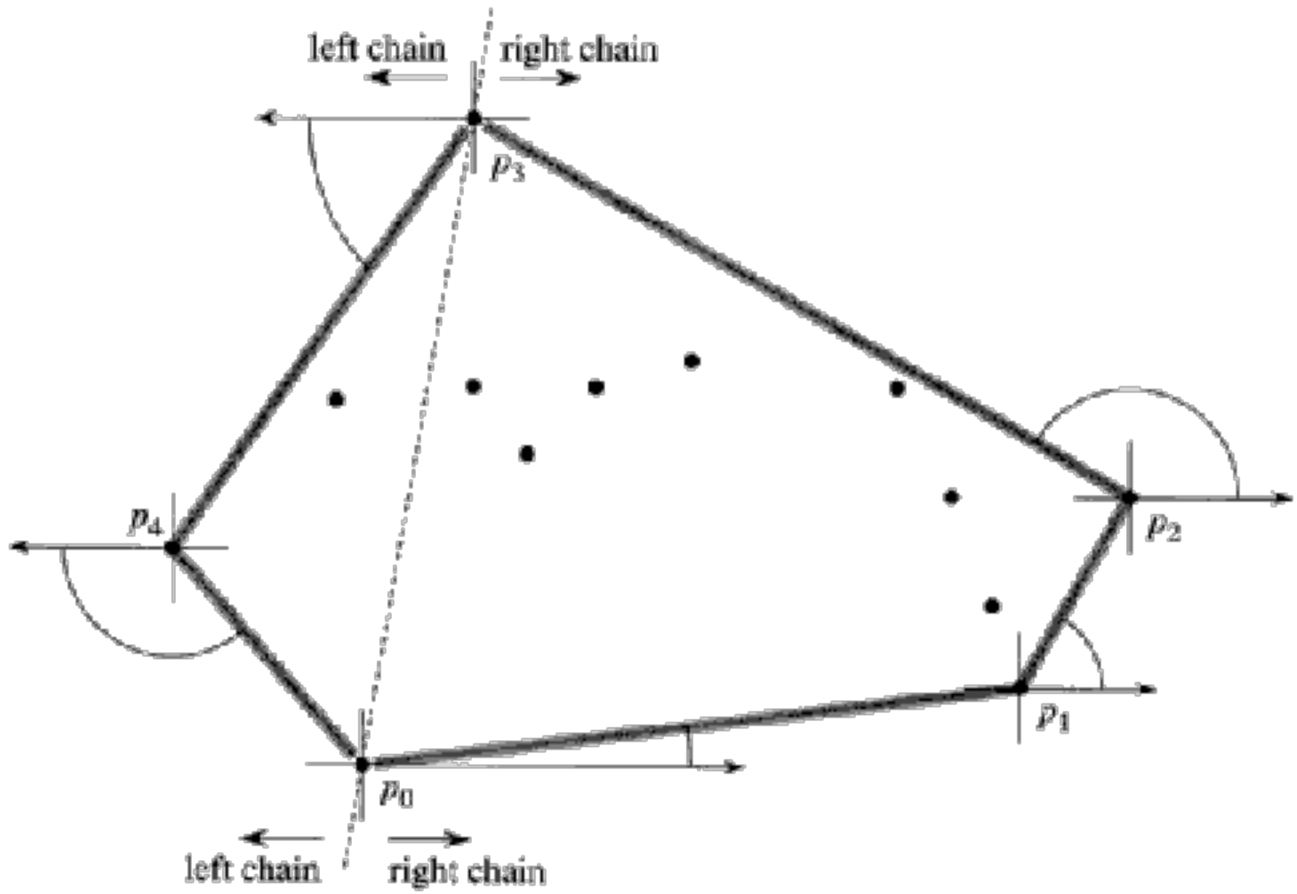
Graham Scan



Convex Hull: Graham Scan applet

- http://www.personal.kent.edu/~rmuhamma/Compgeometry/MyCG/ConvexHull/GrahamScan/grahamScan.htm
  - Main cost: sorting
    - O(n log n)

Package Wrapping: Jarvis March



Package Wrapping: Jarvis March

- Time complexity
  - (Cost of iteration) X (# iterations)
- Each iteration: O(n)
- Number of iterations = O(n)
- Cost = O(nh)
  - h = # of points on convex hull

Complexity of Convex Hull

- Graham Scan: $O(n \log n)$
- Jarvis March: $O(nh)$ [output sensitive]
- Lower Bound = $\Omega(n \log h)$

Other Methods

- Divide and Conquer
- Conquer and Divide
- Randomized algorithms

Chan's Algorithm

- Combines the benefits of both algorithms
- Partition points into n/m groups of size m
- Use Graham scan on each one
  - O((m log m) (n/m)) = O(n log m)
- Merge the n/m convex hulls using a Jarvis march algorithm by treating each group as a "big point"
  - Tangent between a point and a convex polygon with m points can be computed in O(log m) time
  - O((n/m)(log m)(h)) = O((n/m)h log m)

Chan's Algorithm

- Time Complexity = O(n log m + (n/m) h log m)
- If m = h, then time = O(n log h)
- How to guess h?
  - Linear Search
    - Time complexity = O(nh log h)
  - Binary Search
    - Time complexity = O(n $\log^2$ h)
  - Doubling Search (m = 1, 2, 4, 8, …)
    - Time Complexity = O(n $\log^2$ h)
  - ???

Chan's Algorithm: More tricks

- What if $m = h^2$?
  - Then $O(n \log m) = O(n \log h)$
- So try: m = 2, 4, 16, 256, …

$$\sum_{t=1}^{\lg \lg h} n2^t = n \sum_{t=1}^{\lg \lg h} 2^t \le n2^{1+\lg \lg h} = 2n \lg h = O(n \log h),$$

3D convex hulls