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Programming Team

Fall 2024

Preparing for ICPC Competition … 1

- **North America Qualifier (NAQ)**
  - Oct 5, 2-7 PM on Kattis
- Registered: 3 Teams
  - Asymptotic AC; Binary Brains; Ternary Trios
- Link to contest: https://naq24.kattis.com/
- Info at: https://na.icpc.global/naq
- Registration: https://icpc.global/regionals/finder/North-America-Qualifier

ICPC Programming Competition

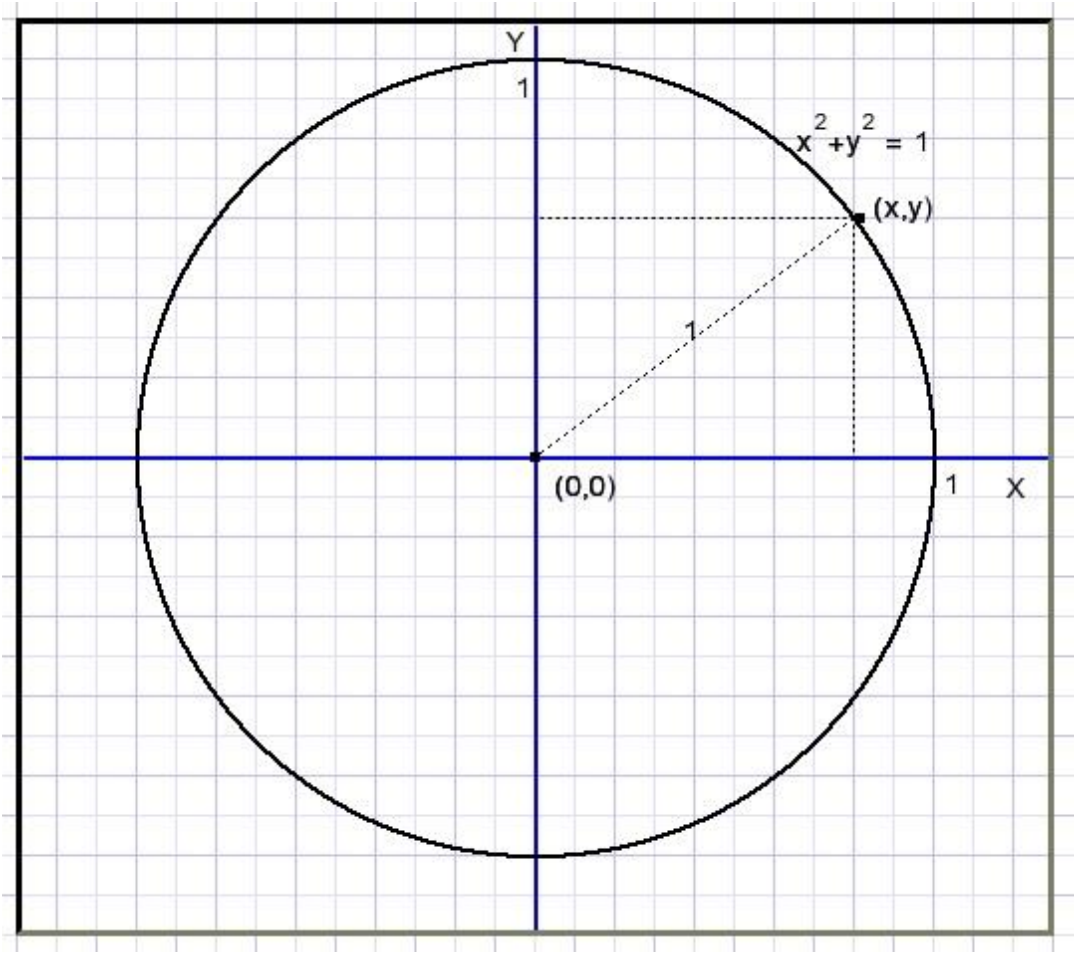
- Nov 16, 2024

QUESTIONS?

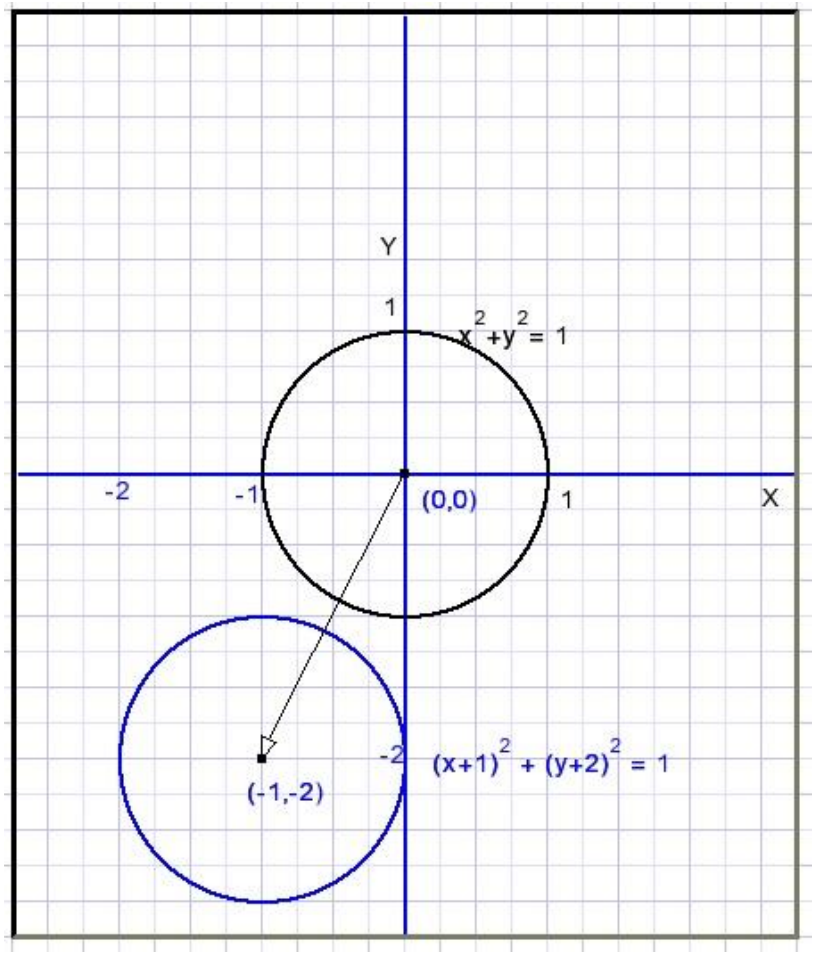
Ellipses

- https://youtu.be/5TQMJ09MLWM (3:09 minutes)
- https://www.davdata.nl/math/ops-on-ellipses.html

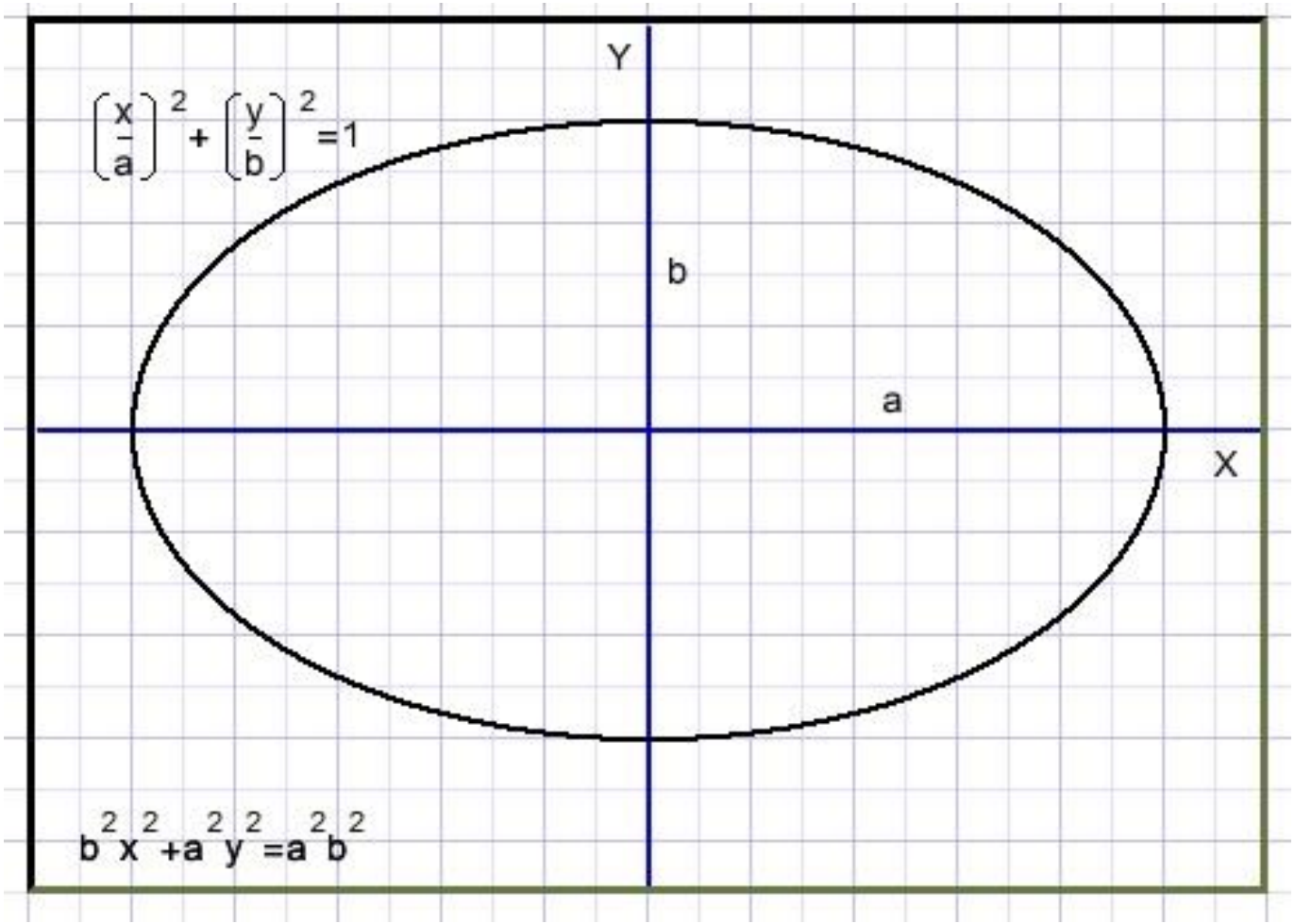
Circle



Translation



Circle to Ellipse: Scaling the axes

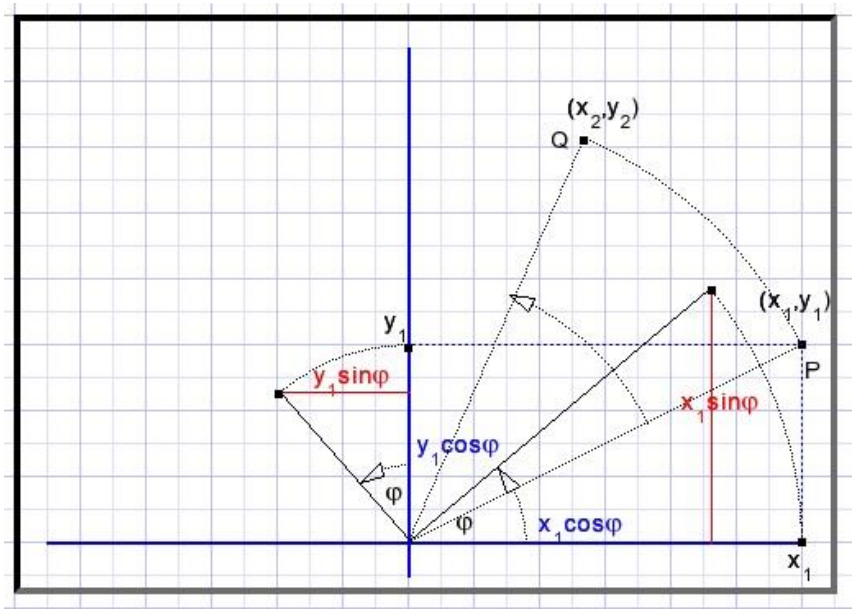


General form: $b^2x^2 + a^2y^2 = a^2b^2$

Translated Ellipse

- Centered at origin along axes: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- Center translated to (h, k): $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
- General form: $b^2x^2 + a^2y^2 + px + qy + s = 0$

Rotations clockwise by ϕ

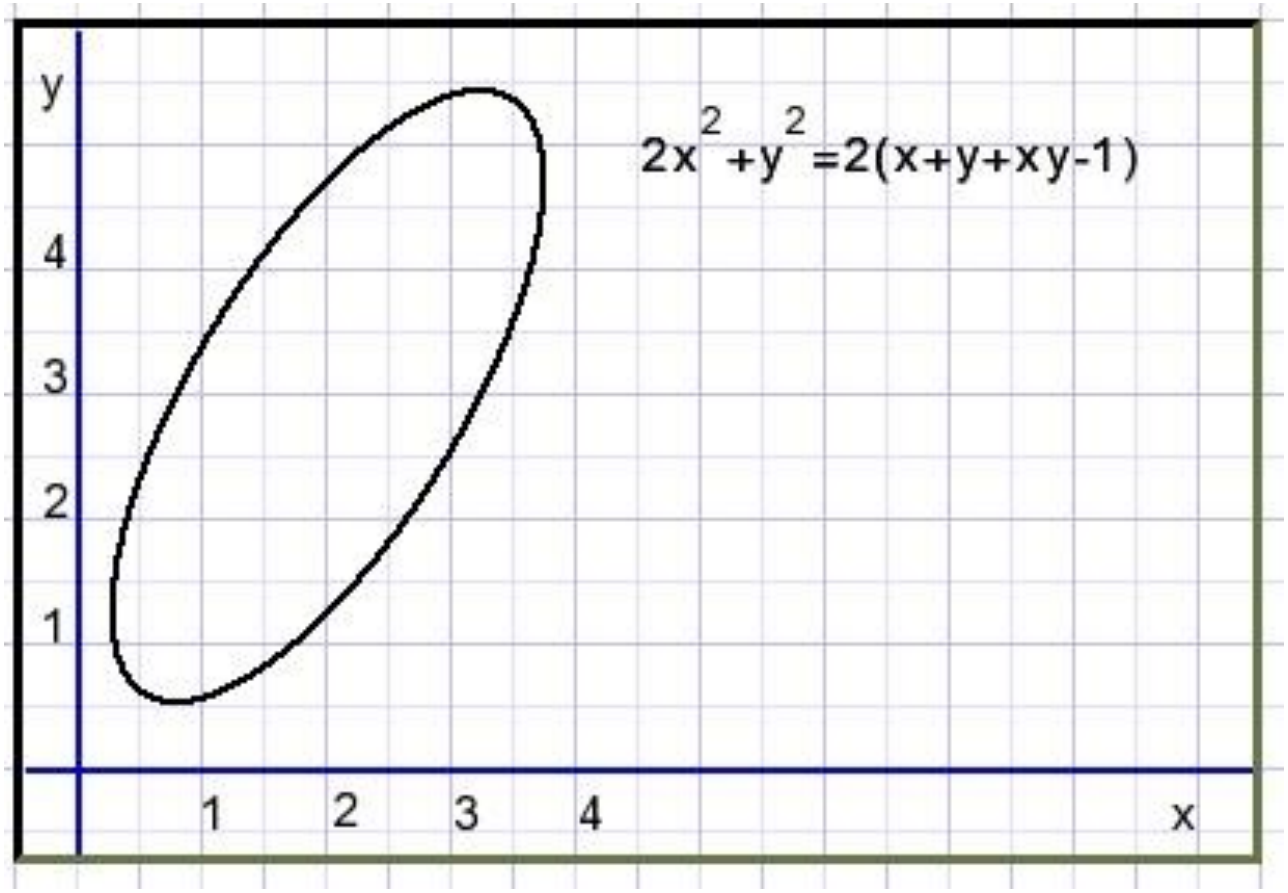


- Rotated by ϕ deg cw at (0,0)
- Rotation of $(x_1,0)$ causes
  - New $x = x_1\cos\phi$
  - $\Delta y = x_1\sin\phi$ … infuence of x on y
- Rotation of $(0,y_1)$ causes
  - New $y = y_1\cos\phi$
  - $\Delta x = y_1\sin\phi$ … influence of y on x
- $x_2 = x_1\cos\phi - y_1\sin\phi$
- $y_2 = y_1\cos\phi + x_1\sin\phi$
- x is replaced by $x\cos\phi - y\sin\phi$
- y is replaced by $y\cos\phi + x\sin\phi$

Ellipse Equation

- Centered at origin along axes: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
- Center at (h,k): $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
- Rot.: $\frac{(x\cos\phi-y\sin\phi)^2}{a^2}+\frac{(y\cos\phi+x\sin\phi)^2}{b^2}=1$
- $\frac{(x\cos\phi-y\sin\phi-h)^2}{a^2}+\frac{(y\cos\phi+x\sin\phi-k)^2}{b^2}=1$
- Slope of major axes$=\tan\phi$
- Translation: x,y terms; Rotation: xy term

Rotated Ellipse



General form: $b^2x^2 + a^2y^2 + px + qy + rxy + s = 0$

To find extreme points

- Start with ellipse equation: $f(x, y) = 0$
- For vertical extreme points:
  - Differentiate f with respect to x and set y' to 0.
  - Solve for x and y
- For horizontal extreme points:
  - Differentiate f with respect to y and set x' to 0.
  - Solve for x and y
- E.g., $f: b^2x^2 + a^2y^2 + px + qy + rxy + s = 0$

Remaining challenge

- How to arrive at the ellipse equation, given:
  - Coordinates of the two foci
  - Length of the major axis
- Case 1: If foci are along x- or y-axis & centered at origin
- Case 2: If foci are along x- or y-axis but not centered at origin
- Case 3: If foci are not on either axis, but centered at origin
- Case 4: General case

General Case

- Given:
  - Coordinates of the two foci
  - Length of the major axis
- Find center, lengths of major and minor axes
- Find slope of major axis, and rotation angle
- Write down equation and simplify it
- Find extreme points as described