Introductory Tutorial

Place/Transition Nets I

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I. Introduction to place/transition nets
II. Basic analysis techniques

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I. Introduction to place/transition nets

An example
- Features of PT-nets
- PT-nets vs EN-systems

Formal definitions
- PT-net
- Occurrence sequence, reachability
- Marking graph

Behavioral properties
- Deadlock, Liveness
- Boundedness, 1-safety
- Reversibility

Extensions
- Capacities
- Complement places
- Inhibitor arcs

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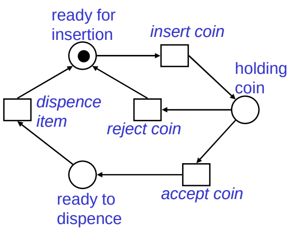
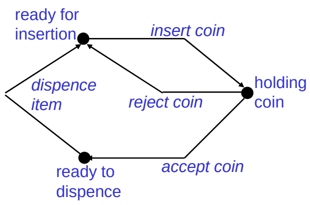
Example: a vending machine

Control structure:





an EN system

its behaviour

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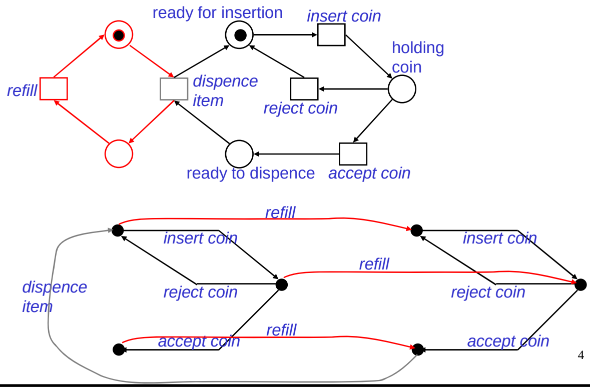
Example: a vending machine

Adding concurrent refill – capacity one



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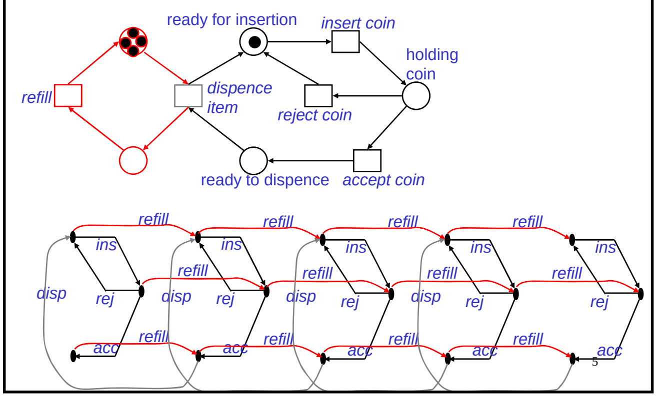
Example: a vending machine

Adding concurrent refill – capacity **four**



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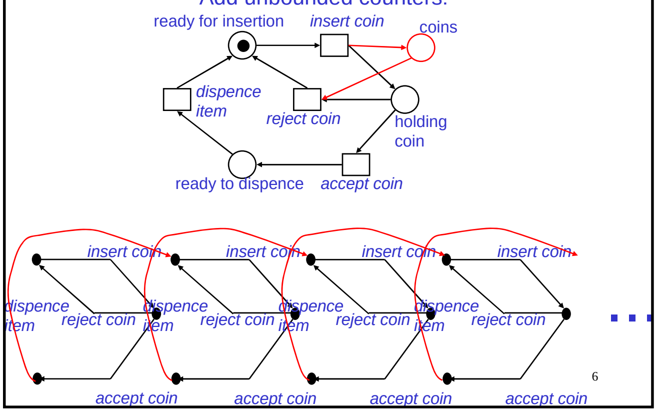
Example: a vending machine

Add unbounded counters:



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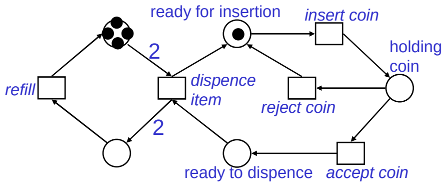
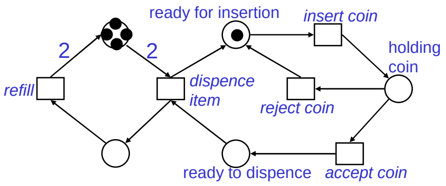
Example: a vending machine

Adding arc weights:



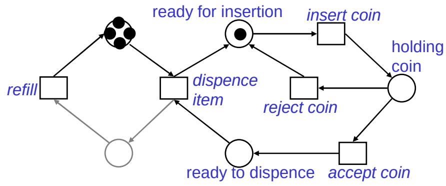
selling pairwise

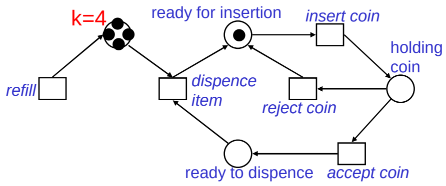


storing pairwise

Example: a vending machine

Adding capacities:





P/T Nets generalize EN systems

Each contact free EN system is a 1-safe marked PT net

Terminology

in EN systems:	in P/T nets
condition	place
event	transition
case, state	marking
$c \subseteq$ conditions	m : places → {0,1,....}
sequential case graph	marking graph (reachability graph, state graph)

Formal definition

A **place/transition net** consists of:

S – set of **places**, [german: "Stellen"], finite
T – set of **transitions**, finite, disjoint to S
F – **flow relation**, $F \subseteq (S \times T) \cup (T \times S)$

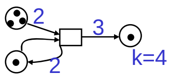
"[S,T,F] is a **net**"

k – partial **capacity** restriction, k: S → {1,2,3,...} ∪ {∞}
w – **arc weight** function, w: F → {1,2,3,...}
m_0 - the **initial marking**, m_0: S → {0,1,2,...} s.t. $\forall s \in S$, $m_0(s) \le k(s)$

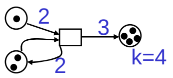
"a **marking**"

$[S,T,F,k,w,m_0]$

Occurrence Rule



$m \xrightarrow{t} m'$
m [t > m'



Transition t is **enabled** at marking m if

for $[s,t] \in F$: $w(s,t) \le m(s)$ and
for $[t,s] \in F$: $m(s) + w(t,s) \le k(s)$

Successor marking:

$$m'(s) = \begin{cases} m(s) & \text{if } [s,t] \notin F \;\; [t,s] \notin F \\ m(s) - w(s,t) & \text{if } [s,t] \in F \;\; [t,s] \notin F \\ m(s) + w(t,s) & \text{if } [s,t] \notin F \;\; [t,s] \in F \\ m(s) - w(s,t) + w(t,s) & \text{if } [s,t] \in F \;\; [t,s] \in F \end{cases}$$

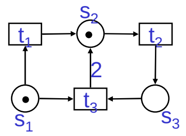
Occurrence sequences, reachability

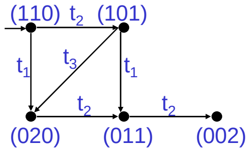
$m_0 \xrightarrow{t_1} m_2 \xrightarrow{t_2} \;......\; \xrightarrow{t_n} m_n$

→ "$t_1\, t_2 \ldots t_n$" is **finite occurrence sequence**
→ m_n is **reachable** from m_0
→ **$[m_0>$** - the set of all reachable markings

$m_0 \xrightarrow{t_1} m_2 \xrightarrow{t_2} \;.......\; \xrightarrow{t_n} \;.....$
→ "$t_1\, t_2 \ldots$" is **infinite occurrence sequence**

Marking graph





Marking graph = directed edge-labeled graph with initial vertex
- vertices = reachable markings
- initial vertex = m_0
- labeled edges = $m \xrightarrow{t} m'$

occurrence sequence = directed path starting at m_0

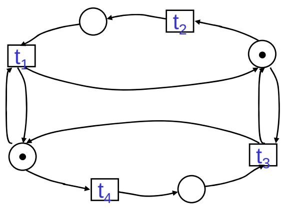
Behavioral properties

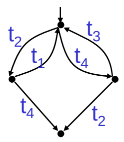
A marked net is

terminating	has only finite occurrence sequences
deadlock-free	each marking enables a transition
live	each reachable marking enables an occurrence sequence containing all transitions
bounded	each place has a bound b(s): m(s) ☯ b(s), for all reachable markings m
1-safe	b(s) = 1 is a bound for all places
reversible	always possible to return to m_0

Our vending machines are deadlock-free and live.
We had 1-safe, bounded, and unbounded versions.
The bounded vending machines are reversible.

Further examples



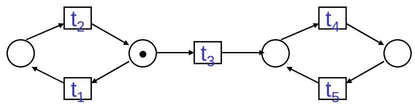


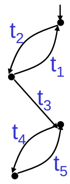
→ not deadlock-free

deadlock = vertex without successor

deadlock-free → not terminating

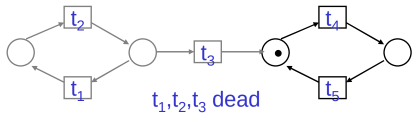
Further examples



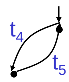


deadlock-free, not live

live = no reachable marking where a transition is dead
(cannot become enabled again)



t_1,t_2,t_3 dead



Boundedness

bounded = finitely many reachable markings

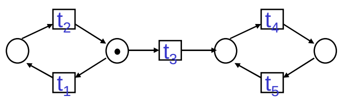
Why?

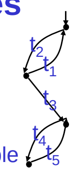
"←" finitely many reachable markings
→ take max. number of tokens as bound

"→" bounded
→ m(s) between 0,....,b(s) → b(s) + 1 possibilities
→ max. $(b(s_1)+1)\ (b(s_2)+1)\ ...(b(s_n)+1)$ different markings
→ finitely many

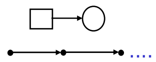
→ 1-safe net has up to 2^n reachable markings

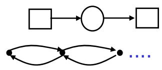
Further examples



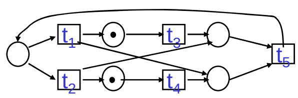


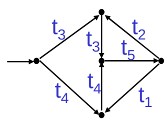
1-safe, deadlock-free, not live, not reversible





unbounded, not reversible unbounded, reversible



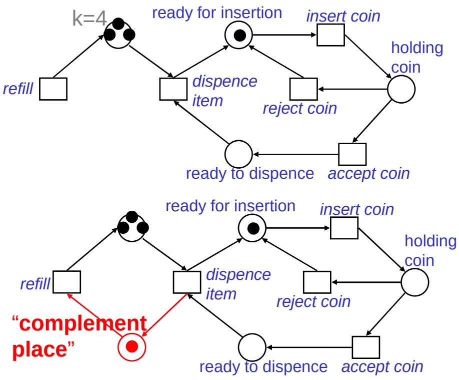


1-safe, live, not reversible

→ reversible = marking graph strongly connected

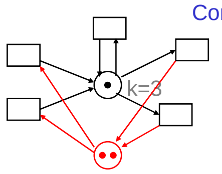
Substituting capacities

Every net with capacities can be replaced by one without!



Weak capacities

Construction:



... does not quite implement original enabling rule, but:

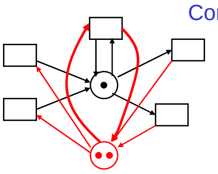
t enabled at m if

- m(s) ❍ w(s,t)	for [s,t] ∈ F	[t,s] ∉ F
- m(s) + w(t,s) ☯ k(s)	for [s,t] ∉ F	[t,s] ∈ F
- m(s) – w(s,t) + w(t,s) ☯ k(s)	for [s,t] ∈ F	[t,s] ∈ F

but: for finite k(s), s is k(s)-bounded

Strong capacities

Construction:



k=3

- implements original enabling rule:

t enabled at m if

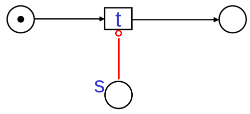
- m(s) ❍ w(s,t)	for [s,t] ∈ F
- m(s) + w(t,s) ☯ k(s)	for [t,s] ∈ F

-generalizes contact in EN systems:

EN system = marked PT net

- no arc weights
- k(s) = 1 (strong!) for all places s

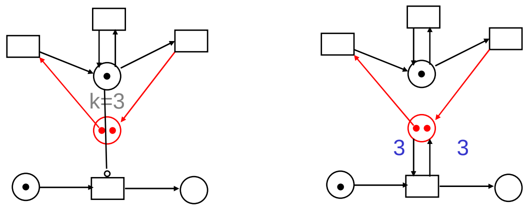
Inhibitor arcs



t only enabled if

m(s) = 0

If k(s) is finite, construction:



II. Basic analysis techniques

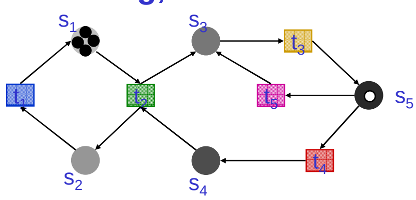
Linear algebra
- Marking equation
- Place invariants
- Transition invariants

Structural tecchniques
- Siphons
- Traps
- Siphon/trap property

Restricted net classes
- State machine
- Marked graph
- Free choice net

Causal semantics
- Occurrence net
- Process net

Marking, transition as vector

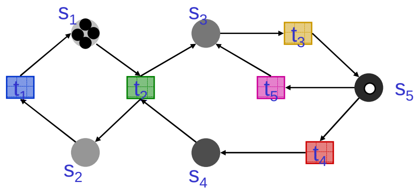


$\underline{m_0}$: (4 , 0 , 0 , 0 , 1)

$\underline{t_2}$ = (-1 , 1 , 1 , 0 , -1)

If $m_0 \xrightarrow{t_2} m_1$ then $\underline{m_0} + \underline{t_2} = \underline{m_1}$ = (3 , 1 , 1 , 0 , 0)

Matrix representation of a net



$$(N) = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & -1 & 0 & 1 & 0 \end{pmatrix}$$

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The marking equation

If $m_0 \xrightarrow{t_2\, t_3\, t_5\, t_1\, t_3} m$ then

$\underline{m}_0 + \underline{t}_2 + \underline{t}_3 + \underline{t}_5 + \underline{t}_1 + \underline{t}_3 = \underline{m}$

$\underline{m}_0 + (1 \cdot \underline{t}_1) + (1 \cdot \underline{t}_2) + (2 \cdot \underline{t}_3) + (0 \cdot \underline{t}_4) + (1 \cdot \underline{t}_5) = \underline{m}$

$\underline{m}_0 + (N) \cdot (1,1,2,0,1) = \underline{m}$

Parikh-Vector of $t_2\, t_3\, t_5\, t_1\, t_3$

→ If $m_0 \xrightarrow{\sigma} m$ then $\underline{m}_0 + (N) \cdot \text{Parikh}(\sigma) = \underline{m}$

→ A marking is only reachable if

$(N) \cdot \underline{x} = (\underline{m} - \underline{m}_0)$ has a solution for nat. $\underline{x}$

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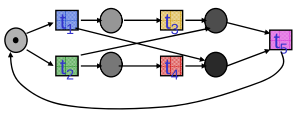
Example



reachable markings:	corresponding solutations
(1 , 0 , 0 , 0 , 0)	(0 , 0 , 0 , 0 , 0), (1 , 0 , 1 , 0 , 1), ...
(0 , 1 , 0 , 0 , 1)	(1 , 0 , 0 , 0 , 0), ...
(0 , 0 , 1 , 1 , 0)	(0 , 1 , 0 , 0 , 0), ...
(0 , 0 , 0 , 1 , 1)	(1 , 0 , 1 , 0 , 0), (0 , 1 , 0 , 1 , 0), ...

non-reachable marking has also solutions! ☹

(0 , 1 , 1 , 0 , 0) (1 , 1 , 0 , 0 , 1), ...

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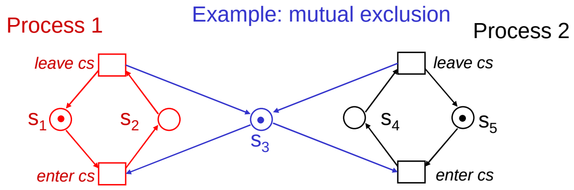
Place invariants

Example: mutual exclusion



mutex: m reachable → $m(s_2) + m(s_4)$ ☯ 1

use place invariant

Proof: 1. $m(s_2) + m(s_3) + m(s_4) = 1$ initially true
2. $m(s_2) + m(s_3) + m(s_4) = 1$ is stable
3. $m(s_2) + m(s_3) + m(s_4) = 1$ → $m(s_2) + m(s_4)$ ☯ 1

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Place invariant i

Def. 1: for all t, $\sum_{[s,t] \in F} w(s) \cdot i(s) = \sum_{[t,s] \in F} w(s) \cdot i(s)$

Def. 2: for all t, $\underline{i} \cdot \underline{t} = 0$

Def. 3: $\underline{i} \cdot (N) = (0,\ldots,0)$

If m reachable from m_0 then $\underline{i} \cdot \underline{m} = \underline{i} \cdot \underline{m}_0$

Proof: $m_0 \xrightarrow{s} m$ → $\underline{m}_0 + (N) \cdot \text{Parikh}(s) = \underline{m}$
→ $\underline{i} \cdot \underline{m}_0 + \underline{i} \cdot (N) \cdot \text{Parikh}(s) = \underline{i} \cdot \underline{m}$
($= 0$)
→ $i \cdot m_0 = i \cdot m$

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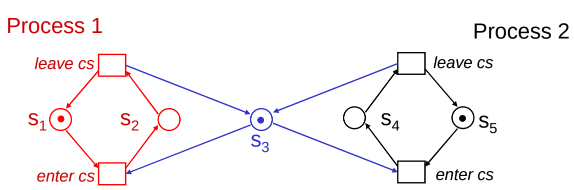
Place invariants



(0 , 1 , 1 , 1 , 0) is place invariant

→ $\underline{i} \cdot \underline{m}_0 = 1 = \underline{i} \cdot \underline{m} = m(s_2) + m(s_3) + m(s_4)$
for all reachable m
→ $m(s_2) + m(s_3) + m(s_4) = 1$ is stable.

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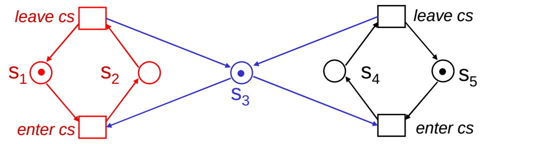
Further place invariants

Process 1

Process 2



(0 , 1 , 1 , 1 , 0) mutual exclusion

(0 , 1 , 1 , 0 , -1) $m(s_2) + m(s_3) = m(s_5)$

→ if s_2 is marked then s_5 is marked

(1 , 1 , 0 , 0 , 0) $m(s_1) + m(s_2) = 1$

→ $m(s_1) \leq 1$, $m(s_2) \leq 1$, s_1,s_2 bounded

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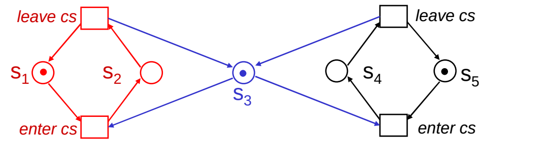
Place invariants and liveness

For all PT nets, -live -no isolated places → for all place invariants $\underline{i}$ - no negative entries - some positive entry s : $\underline{i} \bullet \underline{m}_0 > 0$

(otherwise transitions connected with s are dead)

Process 1

Process 2



(0 , 1 , 1 , 1 , 0) (1, 1 ,0 , 0 , 0) (0 , 0 , 0 , 1 , 1)

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Place invariants and boundedness

if exists place invariant $\underline{i}$ - for all s, i(s) > 0 → net is bounded

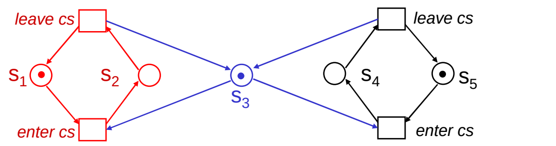
Proof: m reachable → $\underline{i} \bullet \underline{m} = \underline{i} \bullet \underline{m}_0$

→ $i(s) \bullet m(s) \leq \underline{i} \bullet \underline{m} = \underline{i} \bullet \underline{m}_0$

→ $m(s) \leq \underline{i} \bullet \underline{m}_0 / i(s)$

Process 1

Process 2



(1, 2 ,1 , 2 , 1)

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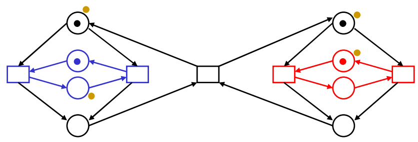
Place invariants and reachability



-m unreachable

-for all place invariants i: $\underline{i} \bullet \underline{m} = \underline{i} \bullet \underline{m}_0$

→ no place invariant is able to prove non-reachability of m

-Marking equation (N) • $\underline{x} = (\underline{m} - \underline{m}_0)$: no solution in naturals

→ marking equation *is* able to prove non-reachability of m

- Marking equation does have rational solution: (1 , 0 , 1 , ½ , ½)

There is a place invariant $\underline{i}$ s.t. $\underline{i} \bullet \underline{m} \neq \underline{i} \bullet \underline{m}_0$ if and only if marking equation does not have rational solution

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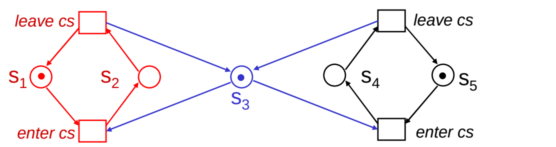
→ *modulo invariants*

Transition invariants

= solutions of (N) • $\underline{y}$ = (0,...,0)

Process 1

Process 2



(1, 1 , 0 , 0) (0 , 0 , 1 , 1) (2, 2 , 1 , 1)

$m_0 \xrightarrow{\sigma} m$ → $m_0 = m$ if and only if Parikh(σ) is transition invariant

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Transition invariants, liveness, boundedness

net - live - bounded → there is transition invariant j - for all t, j(t) > 0

Proof:

By liveness: $m_0 \xrightarrow{\sigma_1} m_1 \xrightarrow{\sigma_2} m_2 \xrightarrow{\sigma_3} m_3$ ....

all transitions all transitions all transitions ....

By boundedness: for some i < j: $m_i = m_j$

→ $m_i \xrightarrow{\sigma_{i+1} \ldots\ \sigma_j} m_j = m_i$

→ Parikh(σ_{i+1} .... σ_j) is transition invariant.

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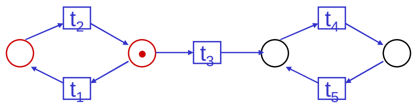
Structural techniques



siphon

once empty – always empty

Def.: $^\bullet S \subseteq S^\bullet$

→if t produces into S, then t consumes from S

trap

once marked, always marked

Def.: $S^\bullet \subseteq {}^\bullet S$

→if t consumes from S, then t produces into S

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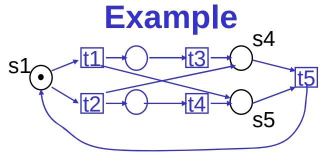
Example



{s1,s4,s5} initially marked trap !

→ (0 , 1 , 1 , 0 , 0) unreachable

(Marking equation could not prove non-reachability!)

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Siphons, traps and liveness, deadlocks

net
- live → every siphon ($\neq \emptyset$) initially marke
- no isolated places

(transitions connected to empty siphon are dead)

net
- has a transition
- no capacity restriction
- all arc weights 1 → net deadlock-free
- every siphon ($\neq \emptyset$) contains initially marked trap

(Set of places unmarked at a deadlock marking is siphon.
This siphon is empty → does not contain marked trap
→ contains no initially marked trap)

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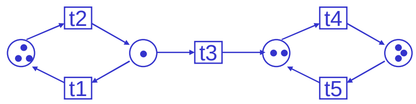
Restricted net classes



-every transition has exactly one pre-place
-every transition has exactly one post-place
-all arc weights 1
-no capacity restrictions

state machine

state machine:

- bounded
- live if and only if - strongly connected, - initially marked

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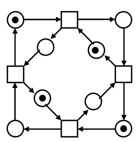
Restricted net classes



-every place has exactly one pre-transition
-every place has exactly one post-transition
-all arc weights 1
-no capacity restrictions

marked graph

marked graph:

- 1-safe if and only if each place belongs to a cycle with exactly 1 token
- live if and only if each cycle initially marked

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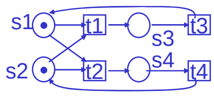
Restricted net classes

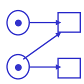
{s1,s3} {s2,s4}
{s1,s3,s4} {s2,s3,s4}
{s1,s2,s3,s4}



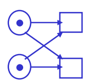
{s1,s2,s3,s4}

-all arc weights 1
-no capacity restrictions
-if transitions share pre-places, they share all their pre-places:
$(s,t) \in F \rightarrow {}^\bullet t \times s^\bullet \subseteq F$

free choice net



not free choice



free choice

free choice is live if and only if every siphon ($\neq \emptyset$) contains initially marked trap

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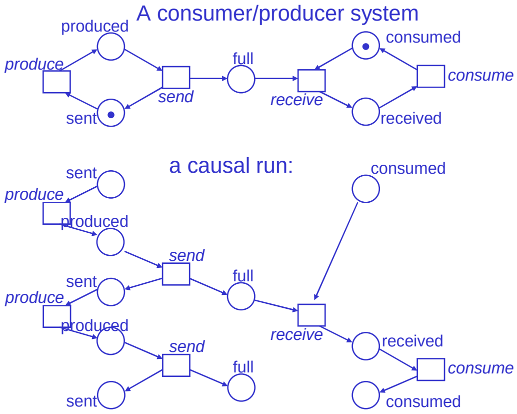
Causal semantics of PT nets



Causal runs

A causal run of a PT net is a labeled Petri net (B, E , K)

net element	name	symbol	interpretation
place	condition	B	token on place
transition	event	E	transition occurrence
arc	causal relation	K	token flow

Occurrence nets

Causal runs:

-no cycles	K+ is partial order ॐ
-no branch at conditions	|b•| ☯ 1, |•b| ☯ 1
-events have finite fan-in, fan-out	•e finite, e• finite
-events have at least one input, one output condition	|•e| ❍ 1, |e•| ❍ 1
-every node has finite "history"	{x | x ॐ n} finite

occurrence net

Process net

represents causal run of a PT net

= occurrence net related to given PT net

labels at B,E

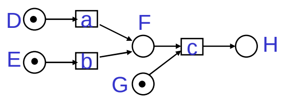
labels: π: B → S, E → T

-m0 agrees with start conditions:
for all s: m0(s) = | {b ∈ B | •b = Ø, π(b) = s} |

-respect transition vicinities
π(•e) = •π(e) , w(s,π(e)) = | {b ∈ •e | π(b) = s} |
π(e•) = π(e)• , w(π(e),s) = | {b ∈ e• | π(b) = s} |

process net

Occurrence sequence vs process net

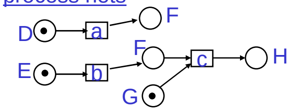


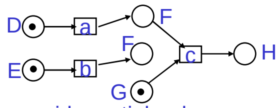
occurrence sequences

abc
bac
acb
bca

provide total orders
respecting causality
independence →
arbitrary interleaving
information about causality
can get lost

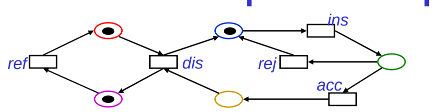
process nets



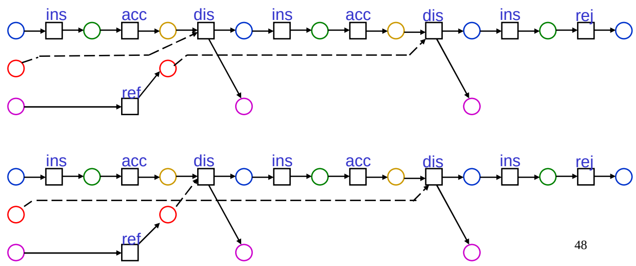


provide partial order
reflecting causality

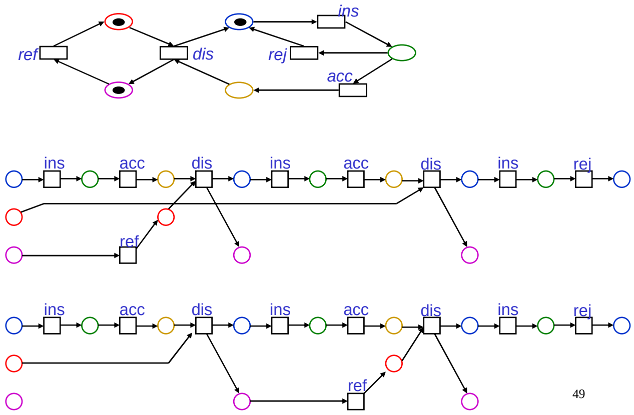
1 occurrence sequence in 2 process nets



ref ins acc dis
ins acc dis
ins rej



2 process nets, no common sequence



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