Minimal Sensor Integrity: Measuring the Vulnerability of Sensor Deployments*

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Abstract

Given the increasing importance of optimal sensor deployment for battlefield strategists, the converse problem of reacting to a particular deployment by an enemy is equally significant and not yet addressed in a quantifiable manner in the literature. We address this issue by modeling a two stage game in which the opponent deploys sensors to cover a sensor field and we attempt to maximally reduce his coverage at minimal cost. In this context, we introduce the concept of minimal sensor integrity which measures the vulnerability of any sensor deployment. We find the best response by quantifying the merits of each response. While the problem of optimally deploying sensors subject to coverage constraints is NP-Complete [3], in this paper we show that the best response (i.e the maximum vulnerability) can be computed in polynomial time for sensors with arbitrary coverage capabilities deployed over points in any dimensional space. In the special case when sensor coverages form an interval graph (as in a linear grid), we describe a better $O(\min(M^2, NM))$ dynamic programming algorithm.

Keywords: Sensor Networks, Game Theory, Sensor Integrity, Graph Algorithms.

1 Introduction

Distributed, real-time sensor networks are essential for effective surveillance in the digitized battlefield and for environmental monitoring. In general, the surveillance zone for the sensors can be viewed as a multidimensional grid with sensors being placed at some of these grid points. Sensors can vary in their monitoring ranges and coverage capabilities of grid points, and have correspondingly different costs. There is a substantial body of literature in sensor networks that addresses techniques for efficient sensor communication [9, 5] and data fusion [8]. With the increasing prevalence of sensor based field operations, research on efficient sensor deployment strategies has also become important [2, 4]. Recently, [3], presented a systematic theory that leads to sensor deployment strategies for effective surveillance and target location. They provide a simplified target location scheme in which every grid point is covered by a unique subset of sensors.

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Given the importance of optimal sensor deployment strategies to battlefield commanders and strategists, the converse problem of reacting to a particular deployment by an enemy is equally significant. In particular, issues related to the vulnerability of different deployment strategies must also be examined. In a battlefield environment, for example, one can naturally expect sensors to be the targets of enemy attacks. To the best of our knowledge, there has been no previous work on quantifying the susceptibility of different placement schemes. In [3], optimal sensor deployment is considered only in the context of coverage and cost constraints while the vulnerability of the deployment has been ignored. Clearly from the deployers perspective, a brute force approach to minimizing grid vulnerability is by maximizing coverage of grid points. However this will unnecessarily increase the deployment cost resulting in inefficient utilization of sensor resources. Thus there is need for a formal framework relating optimal sensor placement to vulnerability.

In this paper, we introduce for the first time the notion of *minimal sensor integrity.* Sensor integrity is a measure of the vulnerability of any sensor placement strategy to attack. Given that the object of any placement strategy is the maximization of a (point) coverage function, the minimal sensor integrity of a placement strategy is the worst case loss of (point) coverage that can be inflicted at least cost.

Our concept of sensor integrity can be better understood from a game-theoretic viewpoint where there are two players: Player 1 deploys M sensors to cover up to N points in a multidimensional grid while satisfying his coverage and cost constraints. Player 2 attempts to destroy sensors based on her removal costs and point *uncoverage* thereby taking into account the tradeoffs between costs and vulnerability. In this paper, we find our best-response to any deployment by player 1.

We can consider two types of point coverages by sensors. In the simple case, the coverage area of a sensor is based on its geographical proximity to the points being sensed. For example, consider sensors equipped with heat or infra-red detection capabilities deployed over a battlefield surveillance zone. Each sensor detects intrusions in the geographical area surrounding it. Thus in general, the total coverage area of a sensor deployment is the intersection of regular polyhedra, each representing the coverage area of an individual sensor (for example, line segments in the 1-D and polygons in the 2-D cases, respectively). In the more general case, the coverage area of a sensor can be arbitrary, i.e., not based on geographical proximity. To illustrate this scenario, consider sensors deployed in battlefields for monitoring and/or jamming enemy RF transmissions emanating from different sites. The given deployment covers these transmission points, however the point coverage area of a sensor depends on the specific frequencies it is monitoring. Thus the overall coverage scheme cannot be represented as an intersection of regular polyhedra.

In this paper, we show that the minimal sensor integrity can be computed in $O(\min(MD^2, ND^2))$ time for sensors with arbitrary coverage over grids of any dimension, where D represents the total coverage by sensors over all points. For the particular case of deployment over a linear grid, we present a dynamic programming solution with a better time complexity of $O(\min(M^2, NM))$ and $O(N + M)$ storage. In this paper, we do not explicitly find player 1's best-response deployment to player 2's actions. However, since we find player 2's best response to every possible deployment of player 1, our technique can be used to identify sensor deployments and removals that form a sub-game perfect Nash equilibrium [11]. Such sequential move games under different deterministic or probabilistic deployment scenarios will be the subject of future research.

2 Sensor Integrity

We begin by formally defining the problem of computing minimal sensor integrity along with a description of the parameters in our model set up. Let $S = \{S_1, S_2, \ldots, S_M\}$ be a set of sensors deployed over a region $G = \{P_1, P_2, \ldots, P_N\}$ of points under any one of a set $\mathcal{T} = \{\mathcal{T}_1, \ldots, \mathcal{T}_r\}$ of possible sensor placement strategies in the given deployment domain. Each sensor placement strategy $\mathcal{T}_i$ is characterized by a given amount of point coverage and has a corresponding deployment cost. For example, one can consider strategies that minimize the cost while satisfying mandated surveillance accuracy parameters. Alternatively, sensors can be placed in such a way as to simplify target location.

Given a placement strategy $\mathcal{T}_i$, the destruction of sensor set $L \subseteq S$ leaves uncovered the set of points $U_L \subseteq G$. We represent the advantage to the opponent of uncovering points in G by a benefit function $B : G \rightarrow \Re^+$. To uncover these points, the opponent pays a sensor removal cost represented by a cost function $C : S \rightarrow \Re^+$.

The *minimal sensor integrity* (MSI) of a given sensor placement strategy $\mathcal{T}_i \in \mathcal{T}$, is defined as:

$$\min_L \left\{ \sum_{S_i \in L} C(S_i) - \sum_{P_i \in U_L} B(P_i) \right\} \quad \forall L \subseteq S. \tag{1}$$

We use the term 'sensor integrity' to refer to the value of the second term in the above equation. The minimization yielding the minimal sensor integrity is carried out over all possible subsets L of S. Thus the set U_L associated with the optimal set of destroyed sensors L in Equation (1) gives the worst case loss of point coverage that can be inflicted at least cost. We denote these optimal sets by U_L^* and L^*.

It is to be noted that equation (1) implicitly assumes an additive mechanism for computing the cumulative costs and benefits of removing and uncovering multiple sensors and points. This assumption need not always be true, for example when the benefits of uncovering adjacent points are correlated. However, in this paper, we solve the MSI problem under the additive assumption. We also assume that sensor placement has been apriori determined using some independent algorithm, for example, one that considers cost and coverage constraints as in [3] and only consider the problem of finding L^* and U_L^* for a given sensor placement strategy.

3 Computing Minimal Sensor Integrity

We consider the problem of computing sensor integrity given a set of M sensors covering a set of N points, with sensor removal cost function C and point uncovering benefit function B. Typically, sensor coverage areas are restricted to be regular polygons. For example, the 2-D problem consists of removing subsets of rectangles or spheres covering a grid. In this and higher dimensional cases, obvious choices of algorithms for computing sensor integrity do not seem to possess either greedy or divide-and-conquer properties. Moreover, the converse problem of optimally deploying sensors subject to coverage constraints is NP-Complete [3] as are the related problems of packing or covering a hyperplane with hyperrectangles [6, 7].

We develop a polynomial time algorithm for minimal sensor integrity by a simple reduction to maxflow on a *directed* bipartite graph. This directed, edge-capacitated bipartite graph $Q = (V_1, V_2, E)$ is constructed as follows: Vertices X and Y act as the source and sink respectively. The set of grid points in G and sensors in S form the other vertices of Q, such that $V_1 = X \bigcup G$ and $V_2 = Y \bigcup S$. The edge set E is defined as follows: There are N directed edges $\{(X, P_1), \ldots, (X, P_i), \ldots, (X, P_N)\}$ assigned flow capacities of $B(P_i)$ each. M directed edges $\{(S_1, Y), \ldots, (S_j, Y), \ldots, (S_M, Y)\}$ are assigned flow capacities of $C(S_j)$ each. Further, for each point $P_i \in G$, $1 \leq i \leq N$, we add outgoing edges (P_i, S_j) directed from P_i to those sensors $S_j \in S$

which cover P_i. The capacity of these edges are set to ∞. Thus this last set of edges ensures that the bipartite graph Q corresponds to the chosen sensor placement strategy.

We reduce the MSI problem to a maxflow problem on Q as follows. Let $L \subseteq S$ be any set of sensors that are destroyed by the enemy. Let $\overline{G} \subseteq G$ be the set of grid points that still remain covered **after** the removal of L, with $U_L = G \backslash \overline{G}$ the set of uncovered points[1]. We use the following notations for simplicity: $(X, \overline{G})$ refers to the set of directed edges in Q from X to vertices in $\overline{G}$, i.e., $\{(X, P_i)\}, \forall P_i \in \overline{G}$, while (L, Y) denotes the directed edges from vertices in L to Y. (X, U_L) and $(S \backslash L, Y)$ are defined in a similar manner. $B(\overline{G})$ denotes the benefit sum $\sum_{P_i \in \overline{G}} B(P_i)$. The terms $B(U_L)$, $C(L)$ and $C(S \backslash L)$ are defined similarly.

The following result shows that every mincut in Q corresponds to an optimal solution of MSI and vice versa.

Theorem 1 *Any arbitrary destroyed sensor set L and associated uncovered points U_L will be the optimal solution to the MSI problem if and only if $(X, \overline{G}) \bigcup (L, Y)$ form a mincut in Q, with corresponding maxflow of $B(\overline{G}) + C(L)$.*

Proof: Assume that L and U_L is an optimal solution for the given MSI problem instance. By Equation (1), the sensor integrity value

$$C(L) - B(U_L) \leq C(L') - B(U_{L'}), \forall L' \subseteq S.$$

Adding $B(G) = \sum_{P_i \in G} B(P_i)$ to both sides, we get

$$C(L) + B(\overline{G}) \leq C(L') + B(G \backslash U_{L'}), \forall L' \subseteq S. \tag{2}$$

Given Equation (2), it now suffices to show that edges $(X, \overline{G}) \bigcup (L, Y)$ form a cut of the directed bipartite graph Q, in order to prove that they also form a mincut. Note that there can be no directed edges in Q from any vertex in U_L to any vertex in $S \backslash L$, since points in U_L become uncovered when sensors in L are removed. There are no directed paths between such vertices either. Hence, removal of the edges $(X, \overline{G}) \bigcup (L, Y)$ will disconnect source X from sink Y. Therefore $(X, \overline{G}) \bigcup (L, Y)$ form a mincut in Q whenever L and U_L is an optimal solution to the MSI problem[2].

For the converse part of the proof, consider an arbitrary mincut in Q. Denote the edges in this mincut by (X, α) and (β, Y), where α and β are the actual vertices in G and S respectively, participating in the mincut. For notational convenience, we relabel α as $\overline{G}$ and β as L respectively. Clearly, no edge of the form (P_i, S_j) can be part of the mincut.

First we need to show that the mincut defines a solution to the MSI problem. To see this, note that there can be no directed paths (and hence no edges) from vertices in $G \backslash \overline{G}$ to vertices in $S \backslash L$ since none of the edges in $(X, G \backslash \overline{G})$ and $(S \backslash L, Y)$ belong in the mincut. Therefore the points corresponding to vertices in $G \backslash \overline{G}$ are uncovered when sensors corresponding to vertices in L are removed. Secondly, every vertex P_i in $\overline{G}$ must have at least one edge directed to some vertex in $S \backslash L$. Otherwise, the edge (X, P_i) is unnecessary in the mincut which is a contradiction. Therefore, the points corresponding to vertices in $\overline{G}$ remain covered when sensors corresponding to L are destroyed. Hence the given mincut of capacity $C(L) + B(\overline{G})$ defines a solution to the MSI problem

[1] Without loss of generality, we assume that every grid point is originally covered by at least one sensor in S. Thus $\overline{G} = G$ if $L = \Phi$. We also assume that every sensor covers at least one point.

[2] It is to be noted that this result relies on graph Q being **directed**. Otherwise, paths traversing vertices from $(X, U_L, L, \overline{G}, S \backslash L, Y)$ in order, will keep Q connected even after removal of edges $(X, \overline{G}) \bigcup (L, Y)$.

with a sensor integrity value of $C(L) - B(G \backslash \overline{G})$. To show that this solution is also optimal, we note from the preceding result that the optimal MSI solution L^* and U_L^* defines a mincut of capacity $C(L^*) + B(G \backslash U_L^*)$. Since all mincuts have the same capacity, we must have

$$C(L) + B(\overline{G}) - B(G) = C(L^*) + B(G \backslash U_L^*) - B(G)$$

and therefore the sensor integrity

$$C(L) - B(G \backslash \overline{G}) = C(L^*) - B(U_L^*)$$

Thus any mincut $(X, \overline{G}) \bigcup (L, Y)$ in Q corresponds to an optimal solution to the MSI problem. □

Figure 1 ilustrates the directed bipartite graph corresponding to the deployment of sensor set $S = \{S_1, S_2\}$ over points $G = \{P_1, P_2\}$, with S_2 covering P_2 and S_1 covering P_1 and P_2. The points have benefits $B(P_1) = 100$ and $B(P_2) = 1$ with sensor removal costs of $C(S_1) = 1$ and $C(S_2) = 100$ respectively. The optimal solution to the MSI problem is to remove S_1 and uncover P_1 which corresponds to the mincut shown in the figure. Note that if the graph were undirected, the reduction in Theorem 1 would not be valid as the optimal MSI solution no longer corresponds to a mincut.

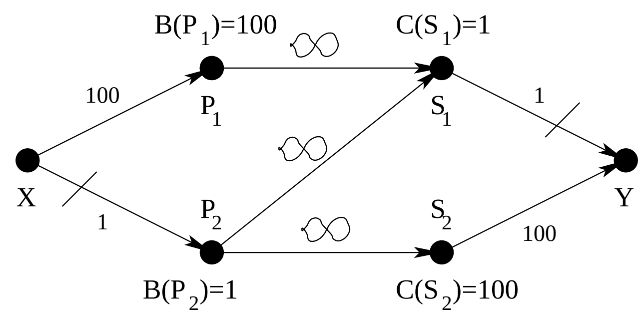

Figure 1: Maxflow reduction for a given sensor deployment.

Using standard maxflow techniques [1], L^* and U_L^* can be computed in $O(\min(M^2E, N^2E))$, where E is the edge set of the bipartite graph Q. Note that this reduction allows us to compute the minimal sensor integrity even while considering sensors of arbitrary ranges and unrestricted (non-polygonal) coverage areas. Hence this allows us to consider situations such as sensors in a 3-D grid monitoring RF transmissions from arbitrary points on specific wavelengths.

In the special case of sensors with linear ranges, we can find a poynomial time solution of much lower complexity by exploiting the order among the sensors. We note that the intersection graph of sensors covering a linear grid forms an interval graph. There are many instances of problems that are more easily solved on interval graphs, for example [10] shows that weighted integrity is polynomial on interval graphs, while it is NP-Complete for comparability graphs.

Let τ_{opt} refer to the optimal value of minimal sensor integrity obtained from equation (1) with L^* and U_L^*. Note that the optimal solution in linear grids posseseses the following property.

Observation 1 *Any optimal solution $\tau_{\text{opt}}^{P_i}$ in which point P_i is uncovered will have value*

$$\tau_{\text{opt}}^{P_i} = \tau_{\text{opt}}^{-P_i} - B(P_i) + \sum_{S_j \in S^{P_i}} C(S_j),$$

where S^{P_i} is the set of sensors that cover P_i and $\tau_{\text{opt}}^{-P_i}$ is the optimal solution computed from $S \backslash S^{P_i}$ and $G \backslash P_i$. The overall optimal solutions over S and G are related to the individual optimal solutions as follows:

$$\tau_{\text{opt}} = \min_{P_i \in G} \{\tau_{\text{opt}}^{P_i}\}.$$

$$U_L^* = \{P_j | \tau_{\text{opt}}^{P_j} = \tau_{\text{opt}}\}.$$

$$L^* = \{\bigcup S^{P_j} | P_j \in U_L^*\}.$$

Unlike in higher dimensional grids, a property of linear grids is that removing a point P_i from G and S^{P_i} from S disconnects both sets, leading to two smaller subproblems. The following result suggests a dynamic programming algorithm for computing minimal sensor integrity in a linear grid by exploiting the order among sensors to eliminate sensors and grid points not contributing to the optimal solution. Consider any set of sensors $S = \{S_1, S_2, \dots S_M\}$, where $S_i = [P_{A_i}, P_{E_i}]$. S is ordered such that $P_{E_1} \leq P_{E_2} \dots \leq P_{E_M}$. Thus each sensor in S corresponds to a closed interval of its coverage points. A_i and E_i are indices representing the beginning point and end point of sensor S_i's coverage, $1 \leq A_i, E_i \leq N$. Let $\tau^l(P_j)$ represent the minimal sensor integrity when considering only grid points in $[P_1 \dots P_j]$, where $j < N$ and sensors $S_1, S_2 \dots S_l$, $1 \leq l \leq M$. Define $\tau^0(P_j) = \sum_{t=1}^{j} B(P_t)$. Note that $\tau^0(P_j)$ merely represents the boundary case of the benefit sum of the first j points and is defined for mathematical convenience. No interpretation may be attached to it.

Let $P_{A_i}^-$ refer to the point immediately preceding point P_{A_i}, $1 \leq A_i \leq N$. Similarly $P_{E_i}^+$ refers to the point immediately following point P_{E_i}, $1 \leq E_i \leq N$. For any sensor S_i, we consider the projection of $P_{A_i}^-$ onto ranges of sensors preceding S_i in S. Let S_l denote the last sensor in S such that either $P_{A_i}^- \in S_l$ or $P_{A_i}^- > P_{E_l}$, $1 \leq l < i \leq M$. If no sensor satisfying these conditions exists, then assign $l = 0$. Then we have the following result.

Theorem 2 *The minimal sensor integrity for sensor set $S' = \{S_1, S_2, \dots S_i\}$, $1 \leq i \leq M$ is given by*

$$\tau_{P_{E_i}}^i = \min\left(0, \tau^{i-1}(P_{E_{i-1}}) + C(S_i) - \sum_{P_{E_{i-1}}^+}^{P_{E_i}} B(j), \left\{\begin{array}{ll} \tau^l(P_{A_i}^-) & \text{if } P_{A_i}^- \in S_l \\ \tau^l(P_{E_l}) - \sum_{P_{E_l}^+}^{P_{A_i}^-} B(j) & \text{if } P_{A_i}^- \notin S_l \\ -\tau^0(P_{A_i}^-) & \text{if } l = 0 \end{array}\right\}\right) \quad (3)$$

Proof: Let $L^* \subseteq S$ be the optimal subset of sensors to be removed for minimal integrity. Consider sensor S_i, the last element of S'. If $S_i \in L^*$ then points $[P_{E_{i-1}}^+, P_{E_i}]$ are uncovered exclusively by removing S_i. This contributes $C(S_i) - \sum_{P_{E_{i-1}}^+}^{P_{E_i}} B(j)$ to the optimal value of sensor integrity. The remaining contribution to the optimal must be $\tau^{i-1}(P_{E_{i-1}})$. Conversely, if $S_i \notin L^*$ then points $[P_{A_i}, P_{E_i}]$ are not included in the optimal uncovering. If $P_{A_i}^-$ intersects the range of any preceding sensors, then the optimal solution is $\tau^l(P_{A_i}^-)$, where l is the last such sensor. Otherwise, the optimal solution is $\tau^l(P_{E_l})$ plus the benefit of removing points from $P_{E_l}^+$ to $P_{A_i}^-$. If no such preceding sensor exists, then the optimal solution only contains the benefits of removing points from P_1 to $P_{A_i}^-$. □

Remark 1 *From 3, note that in addition to the endpoints, the optimal solution must also be computed up to any point within the range of a sensor that just precedes the beginning point of any succeeding sensor. Let $P_j \in S_i$, i.e P_j is a point in the range of sensor S_i. Then,*

$$\tau^i_{P_j} = \min\left(0, \left\{ \begin{array}{ll} \tau^r(P_j) + C(S_i) & \textit{if } P_j \in S_r \\ \tau^r(P_{E_r}) + C(S_i) - \sum_{P^+_{E_r}}^{P_j} B(t) & \textit{if } P_j \notin S_r \\ C(S_i) - \tau^0(P_j) & \textit{if } r = 0 \end{array} \right\}, \right.$$

$$\left. \left\{ \begin{array}{ll} \tau^l(P^-_{A_i}) & \textit{if } P^-_{A_i} \in S_l \\ \tau^l(P_{E_l}) - \sum_{P^+_{E_l}}^{P^-_{A_i}} B(j) & \textit{if } P^-_{A_i} \notin S_l \\ -\tau^0(P_j) & \textit{if } l = 0 \end{array} \right\} \right) \quad (4)$$

where S_r and S_l are the last sensors preceding S_i in S, which either contain or are to the left of P_j

and $P^-_{A_i}$ respectively. The proof is similar to Theorem 2.

To reduce the computation overhead of $\tau^i_{P_j}$, note that the only points of interest at each S_i are the pre-beginning points of succeeding sensors that are within the range of S_i. All such points, along with the optimal solutions at these points can be computed at the time a sensor is first considered for inclusion in the optimal set. For each sensor S_i in the right endpoint ordered set S, define $W^i = \{P^-_{A_i} \bigcup P^-_{A_p} \bigcup P_{E_i}\}$, $\forall P^-_{A_p} \in S_i$, $i+1 \leq p \leq M$. These are the points in S_i, where the optimal sensor integrity must be computed. To compute $\tau^i\{W^i\}$, we need to determine the nearest preceding sensor S_r from S, for each point P_j in W^i. We may also need the sum of benefits from $P^+_{E_r}$ to P_j. To avoid repeated computations, we can precompute and store this term for all such points P_j. This can be done by scanning a sorted list of all points in the grid from left to right while keeping a single running sum of point benefits. This sum is initialized to zero at the beginning and whenever we reach a point that is also a sensor endpoint. The current running sum is stored at each pre-beginning point ($P^-_{A_p}$) that is encountered until the next sensor endpoint is reached.

Algorithm MIN_SENSOR_INTEGRITY

Input:
1. Linear array $G = (P_1, P_2, \dots P_N)$ of grid points.
2. set $S = \{S_1, S_2, \dots, S_M\}$ of sensors covering G, where $S_k = [P_{A_k}, P_{E_k}], P_{A_k} \in G, P_{E_k} \in G, 1 \leq k \leq M$.
3. Benefit function $B : G \rightarrow \Re^+$.
4. Cost function $C : S \rightarrow \Re^+$.

Output: Value = min $(0, C(L^*) - B(U^*_L))$; Optimal set of uncovered points U^*_L; Optimal set of removed sensors L^*.

Preprocessing:
1. Sort G in increasing order.
2. Sort S in non-decreasing order of right end points.

3. Compute running sum of point benefits from each endpoint to all pre-beginning points until the next endpoint.

Procedure:

1. $I_0 = \mathbf{\Phi}$;
2. $U_L^* = [P_1 \ldots P_{A_1}^-]$;
3. `FOR k = 1 to M {`
4. $I_k = I_{k-1} \bigcup S_k$; /* Add S_k to set of sensors considered */
5. $U_L^* = U_L^* \bigcup \{[P_{E_{k-1}}^+ \ldots P_{E_k}]\}$; /* Assume $S_k \in L^*$ */
6. Compute

$$W^k = \{P_{A_k}^- \bigcup P_{A_p}^- \bigcup P_{E_k}\}, \forall P_{A_p}^- \in S_k, k+1 \leq p \leq M.$$

7. $\forall P_j \in W^k$, Compute $S_r : r = \text{ Max}\{q | P_j \in S_q \text{ or } P_j > P_{E_q}\}, S_q \in I_{k-1}$.
8. $\forall P_j \in W^k$, compute $\tau_{P_j}^i$ as in Equation 4.
9. If $\tau^k(P_{E_k})$ implies $S_k \notin L^*$, then $U_L^* = U_L^* - \ \{[P_{A_k}, P_{E_k}]\}$.
10. `}` **End FOR**

Theorem 3 *The value of the minimal sensor integrity is* $\min(\tau^M(P_{E_M}) - \sum_{P_{E_M}^+} B(j))$ *and can be computed in* $O(\min(M^2, MN))$ *time with* $O(M+N)$ *storage. The optimal set of uncovered points is* U_L^* *from which the set of sensors to be removed* L^* *can be calculated.*

Proof: The preprocessing steps in lines 1 and 2 can be completed in $O((N+M)\log(N+M))$ time while the running sums of line 3 can be computed and stored at each point in $O(N)$ time. The For loop in line 3 is executed M times. There are $O(M)$ points in W^k in line 8 for each of which τ values are calculated in $O(1)$ time. Note that if $N < M$, the algorithm can be easily modified to run in $O(NM)$ time by computing the τ values at each point instead of at each sensor. □

4 Conclusions

In this paper we have presented a model that takes into account the costs and benefits of sensor removal and point uncoverage. We have shown that the problem of computing the minimal sensor integrity, i.e., the best response to any sensor deployment is polynomial time solvable. This is in sharp contrast to the sensor deployment problem which is NP-Complete. Furthermore, the algorithm remains polynomial when sensors with arbitrary (non-polygonal) coverage areas are deployed over any dimensional grid.

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