A PARALLEL RANGE SEARCH ALGORITHM USING MULTIPLE ATTRIBUTE TREE

S.V. Nageswara Rao, S.S. Iyengar
Department of Computer Science
Louisiana State University
Baton Rouge, LA 70803, USA

R.L. Kashyap
Department of Electrical Engineering
Purdue University,
West Lafayette, IN 47907, USA

ABSTRACT

The problem of range search arises in many applications in areas such as information retrieval, database, robotics and computational geometry. There are a good number of sequential algorithms for this problem based on data structures such as k-d tree, quad tree, range tree, d-fold tree, super B-tree, overlapping k-ranges, non-overlapping k-ranges, etc. In this paper we present a parallel algorithm for a Single Instruction Multiple Data (SIMD) computing system with p processing elements. We make use of the linearized multiple attribute tree as the underlying data structure. Our algorithm has the complexity of $O(kN/p)$, $p \leq N$ where $k$ is the dimensionality and $N$ is the number of points of the data space.

0. INTRODUCTION

The problem of range search arises in many application areas such as information retrieval, database, robotics, and computational geometry. There have been many sequential algorithms for range search problem using the data structures such as k-d tree, quad tree, range tree, overlapping k-ranges, nonoverlapping k-ranges, d-fold tree, super B-tree, k-fold tree, and multiple attribute tree [1,5-7].

In this paper, we present a parallel algorithm for the range search problem. We make use of the Multiple Attribute Tree (MAT) as the underlying data structure. Our model of computation is a Single Instruction Multiple Data (SIMD) computing system, consisting of $p$ processing elements. The system has a single shared memory that supports simultaneous reads. The time complexity of our algorithm is given by $O(kN/p)$, $p \leq N$.

The organization of this paper is as follows: In Section 1, we discuss the MAT data structure and the corresponding directory. In Section 2 the basic range search algorithm is discussed, and its complexity is estimated. Firstly an $O(k)$ algorithm is obtained using a processor array of size $N$ in a straightforward manner. Then, an $O(kN/p)$, $p \leq N$ algorithm using a processor array of size $p$ and an augmented directory.

1. MULTIPLE ATTRIBUTE TREE AND LINEARIZATION

The MAT data structure was first introduced and analysed by Kashyap et al [2]. The MAT is shown to outperform the inverted file structure for partial match and complete match queries in the cases when the directory resides on the main and secondary memories in [4]. An exhaustive treatment on various structural aspects of MAT can be found in [5].

The k-dimensional MAT on $k$ attributes $A_1A_2...A_k$ for a set of records is defined as a tree of depth $k$, with the following properties [4]:

(i) it has a root at level 0,

(ii) each child of the root is a (k-1)-dimensional MAT, on

(k-1) attributes, $A_1A_2...A_{k-1}$, for the subset of the records

that have the same $A_k$ value. This value is the value of the

root of the corresponding (k-1)-dimensional MAT, and

(iii) the child nodes of the root are in the ascending order of

their values. This set of child nodes is called the fillup set.

From the above definition we note that there is a root node at level 0, and it does not have any value. Every node of level $i$, $i=1,2,...,k$, corresponds to a value of the attribute $A_i$. Fig 1(b) shows the MAT data structure for the sorted set of records of Fig 1(a). The attributes $A_1A_2...A_k$ form the hierarchy of levels 1 through $k$. The nodes of every fillup set are ordered according to their values. Another important property is that each record or point is represented by a unique path from the root to the corresponding terminal node. The total number of nodes in the MAT is almost $kN$ for a given data set containing $N$ records or points.

The MAT is linearized and stored as a directory. We make use of the breadth-first linearization in which the nodes are stored in the order they are encountered when MAT is traversed in a breadth-first manner. The directory is an array of $M$ (SAN) directory elements and each directory element has the following fields:
The idea of breadth-first linearization is illustrated in Fig. 2 and the corresponding directory is shown in Fig. 3. In the first-child field of leaf nodes contain the pointers to the corresponding records. The time complexity of constructing this directory using a uniprocessor system is $O(N \log N)$ (including the time required to sort the records on all the attributes) [5]. Here, we are concerned only with answering the range query, assuming that the directory is available in memory. This directory is utilised in the design of parallel range search algorithms in the next section.

2. PARALLEL RANGE SEARCH ALGORITHM

A range query is given by $Q = \{ q_i \}$, where $q_i$ specifies the range $[l_i, h_i]$ at the level $i$. In geometric terms a range query specifies a rectangle oriented hyper rectangle in $k$ dimensional space. Answering a range query calls for the retrieval of the points enclosed by the specified hyper rectangle, and these points form a "sub" MAT, called the query MAT (QMAT), on the original MAT. These nodes of the QMAT are called the qualified nodes of MAT for the query $Q$. Answering a range query involves identifying the nodes of the QMAT on the MAT. Let $R$ be the number of records contained in the hyper rectangle. The number of the nodes of the QMAT is no more than $Rk$.

An algorithm for range query proceeds by descending down the MAT level by level, starting from the first level. At each level $i$, the child sets of qualified nodes of previous level are searched for the containment in the range $[l_i, h_i]$. The algorithms is as follows:

```
algorithm RANGE-SEARCH(level, qnodes):
begin
1. tempset := ø
2. for each $n \in$ qnodes do
3. add to tempset all the child nodes of $n$ that lie in the range $[l_{level}, h_{level}]$
4. if level < $k$
5. then RANGE-SEARCH(level + 1, tempset)
6. else return the pointers given by the elements of tempset.
end
```

In the algorithm RANGE-SEARCH, as a level $i$, all the nodes that satisfy the partial query $\bigwedge_{j < i} q_j$ are collected as tempset. In the next level $i + 1$, the child nodes of these nodes are tested for inclusion in the range $[l_{level}, h_{level}]$. At the final level, the pointers to the information about the records that satisfy the given query are retrieved.

The MAT data structure has some built-in parallelism with respect to answering a range query. The searching for the qualified nodes can be simultaneously carried out on the sub MATs of the same level. However, there seems to be a stringent sequentiality in the way the attributes are processed one after the other. Again, speaking in geometric terms, processing of each attribute reduces the dimensionality of the search space by one. From the above discussion, we conclude that $O(k)$ is lower bound on the steps involved in answering a range query on the MAT-based approach.
The model of computation is in the form of an array of processor elements \( PE_j, j=1,2,..,p \), and operates in Single Instruction and Multiple Data (SIMD) mode. More specifically, each processor element executes the same algorithm in synchronism with all the other processor elements. We use single shared memory in which the breadth-first MAT directory and other variables are stored. As will be shown later there will be no conflicts in writing into the memory. But, simultaneous read operations are supported. The range query is represented by low-limit\( [i] \) and high-limit\( [i] \), \( i=1,2,..,k \), which give the lower and upper limits of the range for the attribute \( A_j \).

Firstly, consider the processor array \( PE_i, i=1,2,..,N \) where \( N \) is the number of records in input set. The array \( base[i], i=1,2,..,(k+1) \) is stored in the memory, where any node of level \( i \) lies in between the entries indexed by \( base[i] \) and \( base[i]+1 \) in the directory. We use the array \( enable[i], i=1,2,..,N \) to selectively enable and disable the processor elements of the processor array. The algorithm consists of \( k \) steps and in step \( i=1,2,..,k \) the level \( i \) is processed to obtain the qualified nodes of the level \( i \). Each enabled processing element acts on a child node of a qualified node at previous level, and checks if the child node satisfies the range constraint of the current level. The processor elements corresponding to the qualified nodes of current level are enabled in the next step. For the final level, the points that satisfy the range query are obtained. The algorithm executed by the processor element \( PE_j, j=1,2,..,N \) for the step \( i, i=1,2,..,k \) is as follows:

```plaintext
algorithm PE_j;
begin
1. if enable[j]
2. then
3. begin
4. enable[j] = false;
5. \( n = base[i] + j \);
6. if low-limit[i] ≤ value[k] ≤ high-limit[i]
7. begin
8. for \( i = first-child[n] \) to last-child[n] do
9. begin
10. end;
11. end;
end;
end;
```

In the above algorithm the qualified records have to be returned in the final level by suitably modifying the lines 7 and 8. This parallel algorithm implements the algorithm RANGE-QUERY and it is very easily seen that this correctly answers the range query. Since the algorithm is executed in \( k \) synchronous steps, it is evident that the time complexity of this algorithm is \( O(k) \).

In applications involving large amounts of data, the value of \( N \) could be very large and the assumption of the array of \( N \) processors is not very pragmatic. We now present an algorithm that utilizes an array \( PE_i, i=1,2,..,p, p \leq N \) processor elements. The basic idea of 'processing the MAT nodes level by level' is still followed; at any level the processor array is invoked required number of times along breadth of the MAT. In this case the directory is augmented with two more fields - enable and level. The former is used to selectively enable and disable the processor elements and latter field gives the level of the node. The global variable current-level gives the level-number that is currently being processed, and it is incremented after each level is processed. The array index\( [i], i=1,2,..,n \) gives the current index (into the directory) to be used by the processor element \( PE_i \). Initially, the enable field is made true for all the nodes of level 1 and also the entries of the array index corresponding to the first level nodes are filled up. For the final level, the qualified records are retrieved. The algorithm executed by the processor element \( PE_j, j=1,2,..,p \) is as follows:

The if statement in line 2 makes sure that all processors of the SIMD array process the nodes of the current level. After the first \( p \) nodes of a level are processed, the next \( p \) nodes are processed by the use of the array index as in line 8. The lines 6 and 7 are to be modified to retrieve the qualified records in the final level. It is straightforward to see that the range query is processed correctly.
algorithm $PE_j$:
begin
1. $l = \text{index}(j)$;
2. If (($\text{current-level} = \text{level}(l))$ and ($\text{enable}(l)$))
3. then
4. begin
5. if (low-limit[\text{current-level}] $\leq$ value[l])
6. $\leq$ high-limit[\text{current-level}])
7. then
8. begin
9. for $m = \text{first-child}(l)$ to last-child(l) do
10. enable[\text{m}] = true;
11. end
12. index(l) = l + p;
13. enable[l] = false;
14. end;
end;

THEOREM:
The time complexity of the parallel range search algorithm, on an SIMD processor array, if $p$ processors is $O(\sqrt{N/p})$.

PROOF: For the execution of the algorithm, at any level $i$, the maximum number of times the processor array is invoked is given by:

$\lfloor \frac{\text{number of nodes of level } i}{p} \rfloor$

Now, the total number of times the processor array is invoked is given by

$\sum_{i=1}^{k}\left\lfloor \frac{\text{number of nodes of level } i}{p} \right\rfloor p$

$= \sum_{i=1}^{k}\lfloor \frac{\text{number of nodes of level } i}{p} \rfloor p$

$= O(\sqrt{N/p + k})$.

Hence, the time complexity of this algorithm is $O(\sqrt{N/p})$.

We note that for the case $p = N$ this algorithm has the same complexity as the earlier one. But, earlier is superior as it uses less memory space. We also note that the maximum number of times the processor array is invoked is given by $k(\sqrt{N/p} + 1)$. This upper bound corresponds to the worst-case structure of the MAT. In an average-case the number of times that the processor array is invoked will be less than this bound. The author are presently working on this aspect.

3. CONCLUSIONS
We have developed an $O(\sqrt{N/p})$ algorithm for range search problem using the linealized MAT data structure and SIMD computing system. Since the attributes are processed one after the other, the MAT based parallel algorithm has a lower bound complexity of $\Omega(4)$ on the processor array containing no more than $N$ processor elements. Here, we have presented only the order estimates for the time complexity. However, the exact upper bounds and the average-case estimates are needed to further substantiate the importance of this method.

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5. REFERENCES