A Fuzzy Set Approach to Affine Transformation Determination of Point Sets in the Plane

J. M. Zachary *, S. S. Iyengar, and D. H. Kraft

January 2, 1997

Keywords: fuzzy set theory, image understanding, affine transformation, polygonal warping, model-based matching

Introduction and Motivation

Affine transformations describe the rotation, translation, scaling, and shearing of one object relative to another. The transformation

$$\begin{aligned} x' &= ax + by + m \\ y' &= cx + dy + n \end{aligned}$$

maps $\Re^2$ onto itself one-to-one if $ad - bc \neq 0$ and is formally called an *affine transformation* [2]. Thus, three pairs of non-collinear points in the plane determine an affine transformtion. Frequently, in computer graphics and computer vision, one is interested in determining the affine transformation of two objects represented by two point sets in $\Re^2$. However, often, objects in the real world also undergo perspective transformations, which are not affine (*e.g.,* parallelism of lines are not preserved under the transformation). Thus, an exact affine transformation may not exist.

We propose the construction of a fuzzy set of affine transformations in those situations where an exact affine transformation does not exist from a point set P_1 to a point set P_2. The affine transformation with the highest membership value is deemed the best as defined by our membership function. Our

*Contact Author's Address: Louisiana State University Department of Computer Science, Baton Rouge, Louisiana 70803-4020, email: zachary@bit.csc.lsu.edu

derivation improves upon the function proposed in [7] for performing polygonal warping in that we don't rely on external parameterization or angle computations which aren't preserved by affine transformations. We apply our fuzzy set affine transformation framework to model-based matching in object recognition.

Derivation of Fuzzy Membership Function in Polygonal Warping

The fuzzy-based polygonal warping method proposed in [7] is based on interpolating transformation matrices [5]. A membership function for a fuzzy set of possible affine transformations from one point set to another was derived depending upon six explicit parameters [7]. It also required inverse cosine computations for a triangle determined by three non-collinear vertices in a point set. We derive a new membership function which addresses these issues [6] given by

$$smooth_a(v_i^1, v_j^2) = \mathcal{A} * \left(1 - \frac{\mathcal{R}}{180^o}\right) * \mathcal{T} \tag{1}$$

where $v_i^1 \in P_1$ corresponds to $v_j^2 \in P_2$ (here the correspondence is either computed, as in polygonal warping, or is hypothesized, as in object recognition),

$$\mathcal{A} = 1 - \frac{\left|Area_{triangle}(v_{i-1}^0, v_i^0, v_{i+1}^0) - Area_{triangle}(v_{j-1}^1, v_j^1, v_{j+1}^1)\right|}{\max\left(Area_{triangle}(v_{i-1}^0, v_i^0, v_{i+1}^0), Area_{triangle}(v_{j-1}^1, v_j^1, v_{j+1}^1)\right)}$$

determines the degree of equality of the areas of the two triangles determined by $(v_{i-1}^1, v_i^1, v_{i+1}^1)$ and $(v_{j-1}^2, v_j^2, v_{j+1}^2)$ normalized by the area of the larger triangle,

$$\mathcal{T} = 1 - \frac{\max\left(\left|\frac{e_i^0}{e_j^1} - \frac{e_{i-1}^0}{e_{j-1}^1}\right|, \left|\frac{e_i^0}{e_j^1} - \frac{e_{i+1}^0}{e_{j+1}^1}\right|, \left|\frac{e_{i-1}^0}{e_{j-1}^1} - \frac{e_{i+1}^0}{e_{j+1}^1}\right|\right)}{\max\left(\frac{e_{i-1}^0}{e_{j-1}^1}, \frac{e_i^0}{e_j^1}, \frac{e_{i+1}^0}{e_{j+1}^1}\right)}$$

determines the degree of similarity between the triangles $(v_{i-1}^1, v_i^1, v_{i+1}^1)$ and $(v_{j-1}^2, v_j^2, v_{j+1}^2)$ based on the relationship between triangles with equal angles,

$$\frac{e_{i-1}^1}{e_{j-1}^2} = \frac{e_i^1}{e_j^2} = \frac{e_{i+1}^1}{e_{j+1}^2}$$

and $\mathcal{R}$ is the minimum angle of rotation from triangle $(v_{i-1}^1, v_i^1, v_{i+1}^1)$ to triangle $(v_{j-1}^2, v_j^2, v_{j+1}^2)$ as defined by the polar decomposition technique proposed

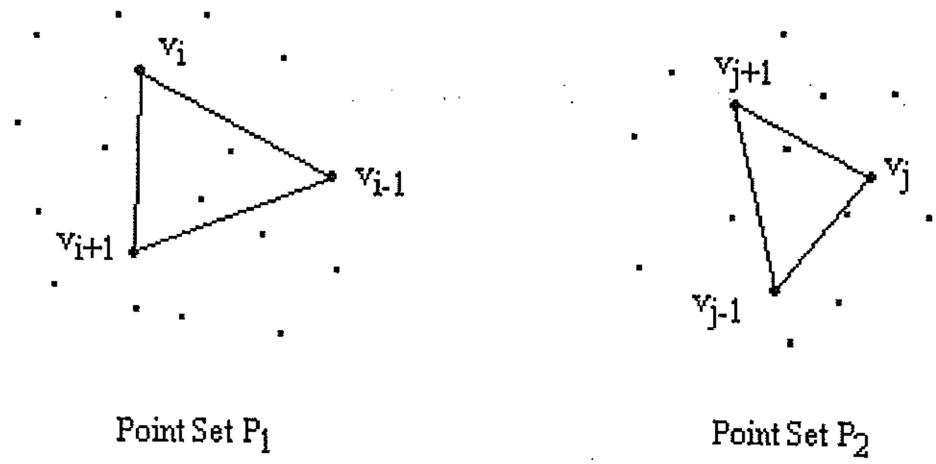


Figure 1: Example triangle pair from two point sets to determine a fuzzy set of affine transformations.

in [5]. Figure 1 is an example of a pair of triangles under consideration. In addition to being self-contained (*i.e.,* not arbitrarily parameterized), our membership function does not require the computation of inverse cosines for angle computation. Preliminary results concur with the theoretical conclusion that our membership function is computationally faster.

Application of Fuzzy Affine Transformations in Model-based Object Recognition

A central problem in computer vision is the identification of objects in an unknown scene. A common approach is the *model-based approach* ([1],[3]) where extracted image features are compared with geometric models of objects. The explicit search for a transformation from the geometric model frame to all or part of some image feature set is called *matching*, and it is a common approach to model-based recognition ([1]).

Images are 2D projections of a world in $\Re^3$, and thus, object features experience perspective projection, a transformation that does not preserve parallel lines, and hence is not affine. Thus, a point set of image features may not have a unique affine transformation to a point set describing a geometric model to which it is compared. We apply our fuzzy set based approach to affine transformations to determine the best affine transformation from one point set to another. This step is useful if the geometric model database to be searched is large compared to the time constraint given for the recognition process. Models which are not sufficiently represented in the fuzzy set can be discarded. The affine transformation fuzzy set is also useful if a "good enough" affine transformation is found without having to employ methods for dealing with other transformations. Thus, our method is well-suited in real-time or near real-time vision applications such as active vision.

Preliminary results with primitive object databases and synthetic image features show this approach is promising. A larger model database to use for real imagery is currently being constructed.

References

[1] R. T. Chin and C. R. Dyer, Model-based recognition in robot vision *ACM Computing Surveys*, 18(1), pp. 67-108, 1980

[2] D. Gans, *Transformations and Geometries*, Appleton-Century-Croft

[3] J. E. Hopcroft, D. P. Huttenlocher, and P. C. Wayner Affine invariants for model-based recognition in *Geometric Invariance in Computer Vision*, eds. Mundy and Zisserman, MIT Press, pp. 354-374

[4] B. K. P. Horn *Robot Vision*, MIT Press

[5] K. Shoemake and T. Duff Matrix animation and polar decomposition *Graphics Interface 92*, pp. 258-264, 1992

[6] J. M. Zachary Interpolating fuzzy affine transformations of points in the plane LSU Department of Computer Science Technical Report

[7] Y. Zhang A fuzzy approach to digital image warping *IEEE Computer Graphics and Applications*, 16(4), pp. 34-41, 1996

INTERNATIONAL PROGRAM COMMITTEE

E. Arce-Medina	National Polytech. Institute	Mexico
R. De Mori	McGill University	Canada
T. Feuring	WWU Muenster	Germany
N. Ishii	Nagoya Institute of Technology	Japan
M. Jamshidi	University of New Mexico	USA
W.W.L. Keerthipala	Nanyang Technological University	Singapore
S.-G. Lee	Hannam University	South Korea
Z.-N. Li	Simon Fraser University	Canada
D. Lukose	University of New England	Australia
N. Mani	Monash University	Australia
D. Martland	Brunel University	U.K.
A.R. Mikler	Ames Laboratory (USDOE)	USA
A.R. Mirzai	Iran University of Science & Technology	Iran
M. Morando	CNR	Italy
D. Mulvaney	Loughborough University	U.K.
N. Serbedzija	GMD FIRST	Germany
A. Sung	New Mexico Tech	USA
S. Valenti	University of Ancona	Italy
H. VanLandingham	Virginia Tech.	USA
J. Wang	Chinese University of Hong Kong	Hong Kong
S. Zhao	Governors State University	USA

IMPORTANT DEADLINES

Abstracts due	January 6, 1997
Acceptance notification	March 3, 1997
Camera-Ready Manuscripts and Registrations	May 2, 1997

IASTED

For more information or to be placed on the mailing list, please contact:

IASTED Secretariat ASC '97
#80, 4500–16 Avenue NW
Calgary, Alberta
Canada T3B 0M6

Tel: 403-288-1195
Fax: 403-247-6851
Email: iasted@cadvision.com
URL: http://www.cuug.ab.ca:8001/~warwodad/iasted.html

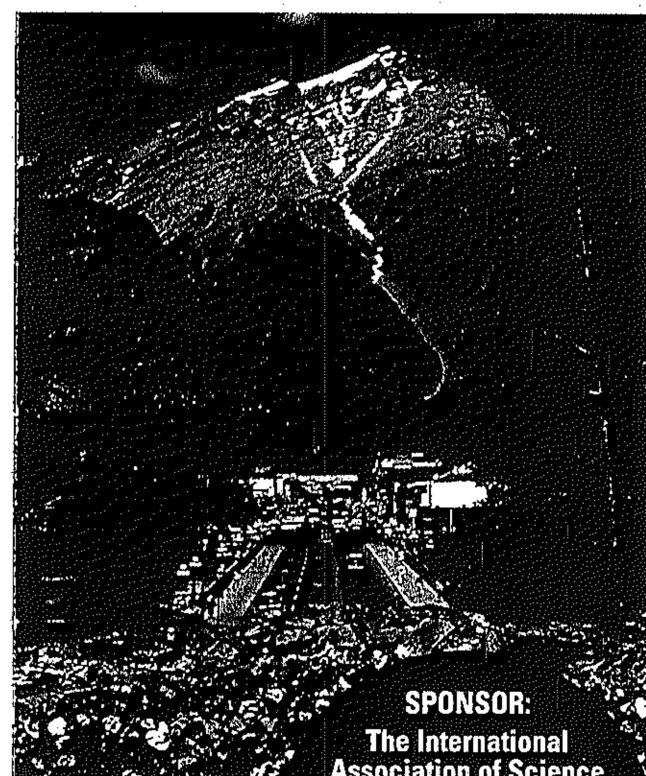
CALL FOR PAPERS

IASTED International Conference

Artificial Intelligence and Soft Computing

July 27 – August 1, 1997

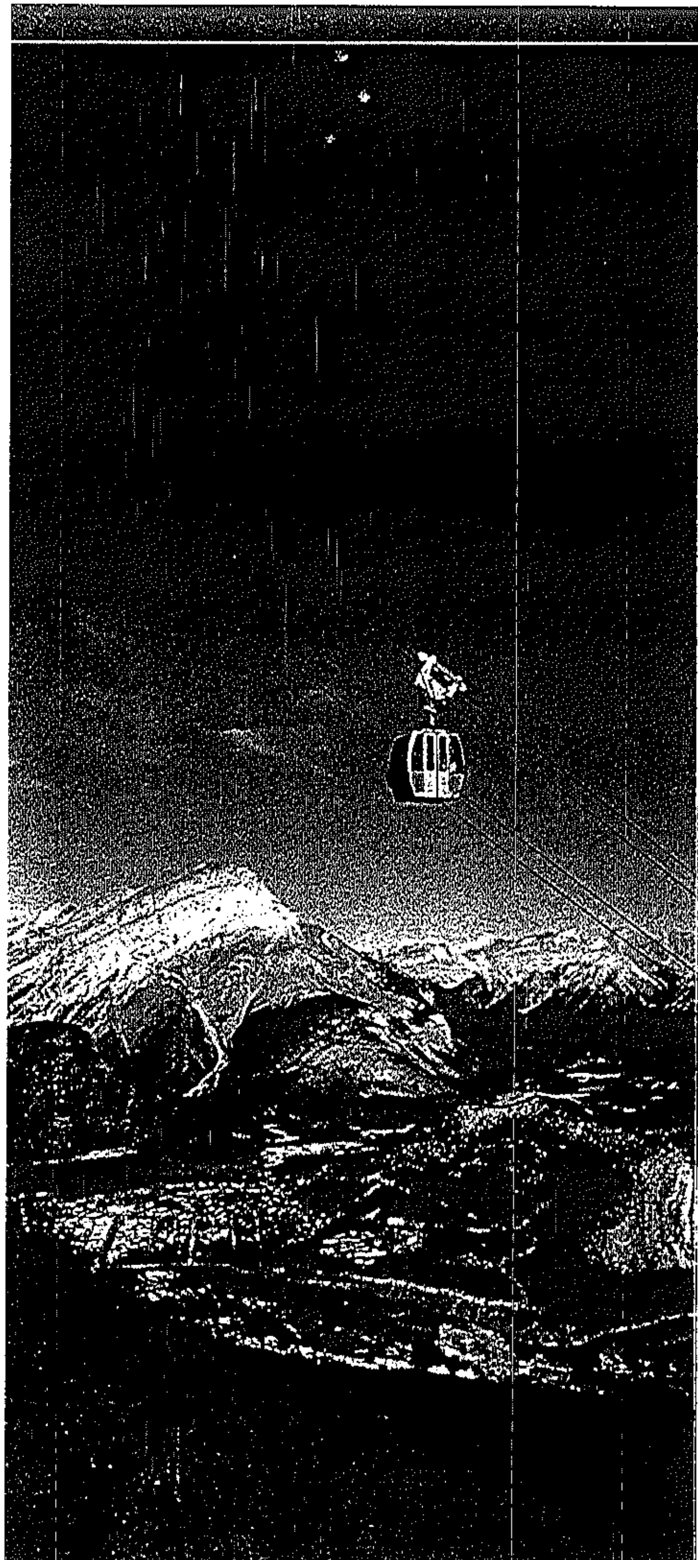
Banff, Canada



SPONSOR:
The International Association of Science and Technology for Development – IASTED

In cooperation with the American Association for Artificial Intelligence – AAAI

IASTED



SCOPE

- Artificial Intelligence
- Expert Systems
- Neural Networks
- Knowledge Acquisition
- Knowledge Representation
- Knowledge Engineering
- Object-Oriented Techniques
- Expert Systems Design Procedures and Tools
- Natural Languages
- Computational Linguistics
- Architectures for Natural Languages
- Algorithms
- Intelligent Databases
- Intelligent Control
- Intelligent Agents
- Distributed Artificial Intelligence
- Image Understanding
- Logic Programming
- Advisory Systems
- User Interface
- Machine Learning
- Fuzzy Expert Systems
- Approximate Reasoning
- Automated Reasoning
- Reasoning Methods
- Temporal and Spatial Reasoning
- Fuzzy Logic
- Fuzzy Systems
- Verification, Validation
- Testing
- Theory of Neural Networks
- Modelling
- Simulation
- Architecture
- Learning Algorithms
- Planning and Scheduling
- Applications
  - Engineering, Science, Business, Economics
  - Power Systems, Transportation, Decision Making
  - Management, Environmental Systems, Banking
  - Government, Education
  - Others

SUBMISSION OF ABSTRACTS

Initial paper selection will be based upon abstracts. An abstract should present a clear and concise view of the motivation of the subject, give an outline of the paper, and provide a list of references. The abstract should not exceed 600 words. The International Program Committee will make the decision concerning the acceptance of the papers.

Three copies of the abstracts should be received at the IASTED Secretariat – ASC'97, #80, 4500 – 16 Avenue NW, Calgary, AB, Canada, T3B 0M6 by **January 6, 1997**. Authors should provide a maximum of five keywords describing their work, and must include a statement confirming that if their paper is accepted one of the authors will attend the conference to present it. Please include the full name, affiliation, full address, telephone number, fax number, and email address of the corresponding author.

Notifications will be mailed by **March 3, 1997.** Authors of accepted papers will be required to submit their camera-ready final papers and registration by **May 2, 1997**. Papers received after that date will not be included in the proceedings. The International Program Committee reserves the right to reject any final manuscript if its quality is not acceptable. If this should occur, the authors will be notified by May 15, 1997 and their registrations will be refunded.