

Minimal Sensor Integrity in Sensor Grids *

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Abstract – *We define the problem of maximal sensor integrity placement, that of locating sensors in n -dimensional grids with minimal vulnerability to enemy attack or sensor faults. We show a polynomial time algorithm for computing sensor integrity exists for sensors with unbounded ranges deployed over a 1-D grid of points. We then present an Integer Linear Programming (ILP) formulation for computing sensor integrity for unbounded range sensors over higher dimension grids.*

Keywords: Algorithms, Sensor field, Reliability, Distributed sensor networks.

1 Introduction

Distributed, real-time sensor networks are essential for effective surveillance in the digitized battlefield and for environmental monitoring. In recent years there have been a number of significant advances in the field of algorithms for effective surveillance using such distributed sensor networks. These advances have ranged from the development of faster algorithms, to the discovery of certain natural problems for which the algorithms fall into the NP-Complete classification.

An important issue in the design of these networks is the optimal placement of sensors in the surveillance zone, also described as the sensor field. In general, the surveillance zone or deployment area for the sensors can be viewed as an n -dimensional grid with sensors being placed at some of these grid points. Sensors can

vary in their monitoring ranges and coverage capabilities of grid points, and have correspondingly different costs. Sensors can either be static or mobile depending on the goals of the overlying application collecting sensor data.

Previous research in distributed sensor networking has largely focused on efficient sensor communication [7, 3] and sensor fusion [1, 6] for a given sensor field architecture. With the increasing prevalence of sensor based field operations, research on efficient sensor deployment strategies becomes correspondingly important. There are several important problems to be solved in this area, given the constraints on sensor capabilities and costs. For example, one can consider optimal sensor deployment strategies that minimize the cost while satisfying mandated surveillance accuracy parameters. Alternatively, sensors can be placed in such a way as to simplify target location. Recently, Chakrabarthy, Iyengar et. al. [2], presented a systematic theory that leads to novel sensor deployment strategies for effective surveillance and target location. They provide coding-theoretic bounds on the number of sensors and present methods for determining their placement in the sensor field.

As research on sensor deployment strategies picks up, issues related to the reliability of different strategies also become important. In a battlefield environment, for example, one can naturally expect sensors to be vulnerable to enemy attacks. To the best of our knowledge, there has been no previous work done on the idea of quantifying the vulnerability of different placement schemes. In [2], optimal sensor deployment

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is considered only in the context of coverage and cost constraints and has been treated independent of the concept of reliability. A brute force approach to minimizing grid vulnerability is by maximizing coverage of sensitive grid points. However, this will unnecessarily increase the deployment cost resulting in inefficient utilization of sensor resources, or will reduce the availability of sensors in other locations. As yet, there is no formal framework in the literature relating optimal sensor placement to vulnerability.

In this paper, we propose for the first time the concept of sensor placement under *maximal sensor integrity*. This notion requires locating sensors in n -dimensional grids with minimal vulnerability. Sensor integrity is a measure of the vulnerability of any sensor placement strategy to enemy attack or sensor faults. Our concept of sensor integrity can be better understood from a game-theoretic viewpoint: for a given sensor placement strategy that satisfies our coverage and cost constraints, the opponent considers a best-response in terms of maximally reducing (damaging) our coverage, at his/her minimal cost. In a simple two-move sequential game, our optimal sensor placement strategy will be the one that maximizes the best-response cost of the opponent. This would correspond to a sub-game perfect Nash equilibrium in game theoretic terms [8]. For a simultaneous move game, the techniques used in [5] can be applied. In a more general, multiple sequential move game, our objective is to determine a set of sequential sensor placement strategies that maximize the best-response costs of the opponent, while assuming limited availability (to us) of replacement sensors. Multiple-move games can also be considered in the context of mobile sensors with the object of minimizing mobile sensor travel costs in successive placement strategies, or damage repair times in the case of quickly reconfigurable and reliable networks. Note that mobility naturally enhances sensor performance and survivability, since mobile sensors can patrol a wide area, can be repositioned for better surveillance, and can even be concealed if vulnerable.

It is likely that the problem of computing minimal sensor integrity for a particular sensor deployment is NP-Complete in the general case of sensors with arbitrary ranges covering grid points. In this paper, we show that a polynomial time algorithm exists for computing sensor integrity for unbounded range sensors deployed over a 1-D grid. We then present an Integer Linear Programming (ILP) formulation for computing sensor integrity for unbounded range sensors over higher dimensional grids.

2 Sensor Integrity

We begin by defining the problem of sensor integrity and then describe our results on its computational complexity. Let $P^* = \{P_1, \dots, P_k\}$ be the set of all possible sensor placement strategies in the given sensor deployment domain. S_k is the set of sensors placed under strategy P_k with G_k the resulting set of grid points covered by S . $U_l \subseteq G_k$ is the set of grid points left uncovered by loss of or faults in sensor set $S_l \subseteq S_k$. $B : G \rightarrow R^+$ is a benefit function representing the advantage to the opponent of uncovering grid points with $C : S_k \rightarrow R^+$, the corresponding sensor removal cost function. Then the optimal sensor placement strategy that maximizes the sensor integrity is defined as:

$$\max_{P^*} \left\{ P_k \in P^* \mid \text{Min} \left\{ 0, \sum_{s \in S_l} C(s) - \sum_{g \in U_l} B(g) \right\} \forall S_l \in 2^{S_k} \right\}, \quad (1)$$

The inside term represents the sensor integrity of the particular placement strategy. The outside term seeks to select the strategy with the maximum sensor integrity.

In this paper, we assume that sensor placement has been apriori determined using some independent algorithm, for example, one that considers cost and coverage constraints [2]. We consider the problem of finding the sensor integrity for a given sensor placement strategy. Our result on determining optimal sensor placement for maximal integrity will be presented in a future version of this paper. We denote the optimal sensor set to be removed by the enemy for minimal remaining integrity by S_{opt} , with G_{opt} the corresponding set of uncovered grid points.

3 Sensor Integrity on a Linear Grid

While the sensor integrity gives us a very elegant way to judge the reliability of a sensor placement in a hostile environment, computationally it is not an easy problem to handle. We believe that the general problem for computing sensor integrity remains NP-complete. The reason of our belief is the similarity of the problem with 0-1 knapsack problem [4]. However, this problem may be polynomial for lower dimensions.

We consider the problem of computing sensor integrity on a linear grid of N points $G = \{P_1, \dots, P_i, \dots, P_N\}$ covered by a set $S = \{S_1, \dots, S_k, \dots, S_M\}$ of M sensors. While the general problem on higher dimensional grids is probably NP-complete, we shall show a polynomial time algorithm to this instance. Our intuition towards

the existence of such a solution is based on the following observations: Consider an arbitrary grid point P_i . Let P_i^- and P_i^+ represent the left and right neighboring points of P_i respectively, and let S^{P_i} be the set of sensors covering P_i . A traditional divide and conquer approach, where we compute optimal solutions from P_0 to P_i^- and P_i^+ to P_N before accounting for P_i is not feasible, as this would entail assigning different costs to sensors in S^{P_i} on both sides. However, we note that removing P_i and S^{P_i} from G and S simultaneously disconnects both sets. While it is not trivial to compute the overall optimal solution from the optimal solutions on either side, we can observe the following:

Lemma 1 *Any optimal solution in which point P_i is uncovered will have value*

$$\tau_{\text{opt}}^{P_i} = \tau_{\text{opt}}^{-P_i} - B(P_i) + \sum_{s \in S^{P_i}} C(s),$$

where $\tau_{\text{opt}}^{-P_i}$ is the minimum value of the optimal solution computed from $S - S^i$ and $G - P_i$.

Proof: We must remove S^{P_i} to uncover P_i and $\tau_{\text{opt}}^{-P_i}$ must be optimal by definition. ■

Lemma 2 *The overall optimal solutions over S and G are related to the individual optimal solutions as follows:*

$$\tau_{\text{opt}} = \text{Min}_{i \in G} \{\tau_{\text{opt}}^{P_i}\}.$$

$$G_{\text{opt}} = \{P_j | \tau_{\text{opt}}^{P_j} = \tau_{\text{opt}}\}.$$

$$S_{\text{opt}} = \{\bigcup S^{P_j} | P_j \in G_{\text{opt}}\}.$$

We expect that τ_{opt} can be computed using Lemma 1 and Lemma 2 in polynomial time. This is because removing P_i from G and S^{P_i} from S disconnects both sets, leading to two smaller subproblems. Note that this property is unique to and can be exploited in linear grids, as can be seen by considering a point and its incident sensors in a 2-D grid.

While Lemma 2 can provide a polynomial time solution, we expect that this approach will be $\Omega(MN^2)$. We therefore present another solution with much lower complexity. First we describe some results to reduce some of the processing, by eliminating unnecessary sensors and grid points not contributing to the optimal solution. Consider an arbitrary sensor $S_k = [P_{B_k}, P_{E_k}]$.

Lemma 3 *If $C(S_k) - \sum_{i=B_k}^{E_k} B(P_i) \geq 0$ then $S_k \notin S_{\text{opt}}$.*

The optimal solution must be computed from $S - S_k$ and $G - [P_{B_k}, P_{E_k}]$.

We are now ready for our main result, which computes sensor integrity by exploiting the order among sensors. Consider any set of sensors $S = \{S_1, S_2, \dots, S_k\}$ ordered as $P_{E_1} \leq P_{E_2} \dots \leq P_{E_k}$. For sensor S_k , let S_p denote the latest sensor in S such that $P_{B_k}^- \in [P_{B_p}, P_{E_p}]$. $p = 0$ if there is no such sensor. Notation $\tau^p(P_i)$ represents the optimal value of the sensor integrity when considering only grid points in $[P_1, P_i]$ and sensors¹ from $S_1, S_2 \dots S_p$. $\tau^0(P_i)$ is defined to be 0 for all P_i . Then we have,

Theorem 1 *The sensor integrity for set S is given by*

$$\tau_{P_{E_k}}^k = \min \left(0, \tau^p(P_{B_k}^-), \tau^{k-1}(P_{E_{k-1}}) + C(S_k) - \sum_{j=P_{E_{k-1}}^+}^{P_{E_k}} B(j) \right). \quad (2)$$

Proof: Let S_{opt} be the optimal subset of sensors to be removed. Consider sensor S_k , the last element of S . If $S_k \notin S_{\text{opt}}$ then points $[P_{B_k}, P_{E_k}]$ remain covered in the optimal solution which must be $\tau^p(P_{B_k}^-)$ by definition. Conversely, if $S_k \in S_{\text{opt}}$ then points $[P_{E_{k-1}}^+, P_{E_k}]$ are uncovered exclusively by removing S_k . This con-

tributes $C(S_k) - \sum_{j=P_{E_{k-1}}^+}^{P_{E_k}} B(j)$ to the optimal value of sensor integrity. The remaining contribution to the optimal must be $\tau^{k-1}(P_{E_{k-1}})$ since S_k is no longer considered. ■

For a given sensor set $S = \{S_1, \dots, S_M\}$ and grid points $G = \{P_1, \dots, P_N\}$, the sensor integrity is given by $\tau_{P_{E_M}}^M$. Note that once $\tau_{P_{E_M}}^M$ is available S_{opt} and G_{opt} can be computed by backtracking. We summarize the algorithm below.

Algorithm MIN_SENSOR_INTEGRITY

Input: Linear array $G = (P_1, P_2, \dots, P_N)$ of grid points; Benefit function $B : G \rightarrow R^+$; Set $S = \{S_1, S_2, \dots, S_M\}$ of sensors covering G where $S_k = [P_{B_k} \dots P_{E_k}]$, $P_{B_k} \in G$, $P_{E_k} \in G$, $1 \leq k \leq M$; A sensor cost function $C : S \rightarrow R^+$.

Output: (Minimum) Value $V = C(S_{\text{opt}}) - B(G_{\text{opt}})$; Optimal set of uncovered points G_{opt} ; Optimal set of removed sensors S_{opt} .

¹Note that sensors are not assigned partial costs, even if only part of their ranges are uncovered.

Procedure:

1. Preprocessing:

- Sort S in non-decreasing order of right end points. $S' = (S_1, S_2, \dots, S_t)$ is the resulting sorted list.
- $S = S' - \{S_i\}, \forall S_i$ such that $C(S_i) \geq \sum_{j=B_i}^{E_i} B(P_j)$. Let $|S| = M$.
- $\forall S_k$: compute $X^k = \{\bigcup P_{B_l}^-, \text{ where } P_{B_l}^- \in [P_{B_k}, P_{E_k}], k+1 \leq l \leq m\}$. Sort X^k in non-decreasing order.

$$X^k = X^k \bigcup P_{E_k}.$$

- $\{\forall i \in X^k \mid i \leq P_{E_{k-1}}\}$: compute $S_i^k = \text{Max}(0, \{p \mid i \in [P_{B_p}, P_{E_p}]\})$, $1 \leq p \leq k$
- $\forall S_k$: compute l_k such that

$$l_k = \begin{cases} k-1 & \text{if } P_{B_k}^- > P_{E_{k-1}} \\ 0 & \text{if } P_{B_k}^- < \text{Min}\{P_{B_p}\} \\ 1 \leq p \leq k-1 \\ \text{Max}\{p \mid P_{B_k}^- \in S_p = [P_{B_p}, P_{E_p}]\} & \\ \text{otherwise} & 1 \leq p \leq k-1 \end{cases}$$

2. Processing:

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 $I_0 = \Phi$  ;
For  $k = 1$  to  $m$  DO
{
 $I_k = I_{k-1} \bigcup S_k$  ; /* Add  $S_k$  to set of sensors considered */
For each point  $i$  in  $X^k$  DO
{
 $\tau^k(i) = \begin{cases} 0 & \text{if } k = 0, \\ \min\{0, \tau^{l_k}(P_{B_k}^-), \tau^{S_i^k}(i) + C(S_k)\} & \\ \text{if } i \leq E_{k-1}, \\ \text{Min}\{0, \tau^{l_k}(P_{B_k}^-), \\ \tau^{k-1}(P_{E_{k-1}}) + C(S_k) \\ - \sum_{j=P_{E_{k-1}}}^i B(j)\} & \\ \text{otherwise.} \end{cases}$ 
}
}

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3. Minimal Sensor Integrity: $= \tau^M(E_M)$.

Theorem 2 Algorithm MAX_SENSOR_INTEGRITY is $O(NM)$.

4 Sensor Integrity for Higher Dimensional Grids

In the previous section, we presented a polynomial time algorithm for computing sensor integrity on a 1-D grid. While the problem remains open for lower dimensions, we present an integer linear programming (ILP) formulation for 2-D grid, which can be extended to higher dimensions as well.

Let us assume that there is a $m \times n$ grid with a sensor placement. In such case, we can extend the cost function of the sensors as follows:

$$C(i, j) = \begin{cases} \text{Cost of destroying the sensor} & \\ \text{if there is a sensor at grid } (i, j), & \\ 0 & \text{otherwise} \end{cases}$$

Further, there is a benefit function $B(i, j)$ for each point of the grid. The benefit function describes the benefit to the enemy if the grid point (i, j) is not covered by any sensor.

In addition, given the sensor placement, there is a 0-1 function called coverage function, $Cov(i, j)$, that describes whether a particular point in the grid is covered by any sensor at all. The formal definition of the coverage function is given below,

$$Cov(i, j) = \begin{cases} 1 & \text{if there is a sensor covering } (i, j), \\ 0 & \text{otherwise} \end{cases}$$

Note that given the sensor placement and information about the ranges of the sensors, $Cov(i, j)$ is known and therefore will not be considered as variables in the final integer linear programming formulation.

For notational convenience, we introduce another 0-1 function $Covers((i, j), (p, q))$ called the sensor specific coverage function which describes whether the grid point (i, j) is covered by a sensor located at grid point (p, q) .

$$Covers((i, j), (p, q)) = \begin{cases} 1 & \text{if a sensor at } (p, q) \\ \text{covers } (i, j), & \\ 0 & \text{otherwise} \end{cases}$$

Note that the sensor specific coverage function is also known. Further, the general coverage function $Cov(i, j)$ may be expressed in terms of the sensor specific coverage function as follow.

$$Cov(i, j) = \max_{(p, q)} Covers((i, j), (p, q)).$$

For the final integer linear programming formulation, we need the following two sets of 0-1 variables.

The first set of variables describe the set of sensors those may be destroyed by the enemy in a least profitable way. By 'least profitable' we mean that the cost of such destruction minus the benefit obtained from the destruction will be maximum possible or equal to the sensor integrity of that particular sensor placement. For our purpose, i.e., for computing the sensor integrity of the sensor placement, this is the set of variables we are naturally interested. Let us call these variables $D(i, j)$. The formal definition of this set of variables is given below.

$$D(i, j) = \begin{cases} 1 & \text{if a sensor at grid } (i, j) \\ & \text{is to be destroyed,} \\ 0 & \text{otherwise} \end{cases}$$

There are mn such variables.

In addition, we are interested in a set of mn artificial variables. These set of variables describes whether a particular is covered after destruction of the sensors described by $D(i, j)$. While these are not independent variables in the sense that given $D(i, j)$ and sensor data, this information is uniquely defined, for the sake of ease of ILP formulation, we shall treat them as independent variables. Further, we shall provide necessary constraints so that these variables may not assume any value inconsistent with $D(i, j)$. We call these set of variables $DCov(i, j)$. The formal definition of $DCov(i, j)$ is,

$$DCov(i, j) = \begin{cases} 1 & \text{if there is no sensor covering } (i, j) \\ & \text{after destruction,} \\ 0 & \text{otherwise} \end{cases}$$

To make sure that after destruction, the set of destroyed sensors gives us the sensor integrity, we choose the objective function of our ILP as

$$\max \sum_{(i,j)} C(i, j)D(i, j) - \sum_{(i,j)} B(i, j)DCov(i, j).$$

Clearly, a set of consistent values maximizing the objective function will give the sensor integrity.

The objective function is to be maximized subject to the following constraints.

1. The standard 0-1 constraint

$$D(i, j) \in \{0, 1\}, DCov(i, j) \in \{0, 1\},$$

$$\text{for all } i = 1, \dots, m \text{ and } j = 1, \dots, n$$

2. A sensor may be destroyed only if there is sensor at that point. Hence,

$$D(i, j) \leq C(i, j),$$

$$\text{for all } i = 1, \dots, m \text{ and } j = 1, \dots, n$$

3. If there is a sensor covering (i, j) after destruction, then $DCov(i, j)$ will have to be 0.

$$DCov(i, j) \leq Covers((i, j), (p, q))D(p, q),$$

$$\text{for } 1 \leq i, p \leq m \text{ and } 1 \leq j, q \leq n$$

4. If there is no sensor covering (i, j) after destruction, then $DCov(i, j)$ will have to be 1.

$$DCov(i, j) \geq \sum_{(p,q)} Covers((i, j), (p, q))D(p, q)$$

$$- \sum_{(p,q)} Covers((i, j), (p, q)) + 1, \forall i, j.$$

Now the sensor integrity of that particular sensor placement may be approximated by using well known heuristics to solve the 0-1 ILP.

5 Conclusions

In this paper, we have defined the concept of sensor integrity as applied to sensor grids. We have shown the existence of a polynomial time algorithm for computing sensor integrity, given unbounded range sensors deployed over a 1-D grid. We then describe an Integer Linear Programming (ILP) formulation for computing sensor integrity for unbounded range sensors over higher dimensional grids.

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