Sensor Placement in Distributed Sensor Networks using a Coding Theory Framework[1]

Krishnendu Chakrabarty
Dept. Electrical & Computer Engineering
Duke University, Durham, NC 27708
krish@ee.duke.edu

S. S. Iyengar
Dept. Computer Science
Louisiana State University, Baton Rouge, LA 70803
iyengar@bit.csc.lsu.edu

1 Introduction

An important issue in the design of distributed sensor networks is the optimal placement of sensors for target location. If the surveillance region, also referred to as sensor field, is represented as a grid (two- or three-dimensional) of points (coordinates), target location refers to the problem of pin-pointing a target at a grid point at any point in time. For enhanced coverage, a large number of sensors are typically deployed in the sensor field, and if the coverage areas of multiple sensors overlap, they may all report a target in their respective zones. The precise location of the target must then be determined by examining the location of these sensors. Target location can be simplified considerably if the sensors are placed in such a way that every grid point in the sensor field is covered by a unique subset of sensors.

The sensor placement problem for target location is closely related to the alarm placement problem [1], which refers to the problem of placing "alarms" on the nodes of a graph G such that a single faults in the system can be diagnosed. The alarms are therefore analogous to sensors in a sensor field. It was shown in [1] that the alarm placement problem is NP-complete for arbitrary graphs. However, for restricted topologies, e.g. a set of grid points in a sensor field, a coding theory framework can be used to efficiently determine sensor placement. The sensor locations correspond to codewords of an identifying code [2] constructed over the grid points in the sensor field.

2 Sensor placement for target location

The identifying code problem can be stated as an optimal covering of vertices in an undirected graph G such that any vertex in G can be uniquely identified by examining the vertices that cover it. A *ball* of radius r centered on a vertex v is defined as the set of vertices that are at distance at most r from v. The vertex v is then said to cover itself and every other vertex in the ball with center v. The formal problem statement is as follows: Given an undirected graph G and an integer $r \geq 1$, find a (minimal) set C of vertices such that every vertex in G belongs to a unique set of balls of radius r centered at the vertices in C. The set of vertices thus obtained constitutes a code for vertex identification.

The problem of placing sensors for unique target identification can be solved using the theory of identifying codes. The grid points in the sensor field correspond to the vertices in the graph G, while the centers of the balls correspond to the grid points where sensors are placed. The unique identification of a vertex in G corresponds to the unique location of a target by the sensors in the sensor field. Each sensor at a grid point can detect a target at grid points that are adjacent to it.

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Let S_n^p denote the number of sensors required for uniquely identifying targets in an n-dimensional ($n \leq 3$) sensor field with p grid points in each dimension. The following theorem provides upper and lower bounds on S_n^p.

Theorem 1 *The number of sensors S_n^p for uniquely identifying a target in an n-dimensional sensor field with p grid points in each dimension is given by: $p^n/(n+1) \leq S_n^p \leq p^n/n$.*

Next, for every grid point (x, y, z) in a sensor field, we associate a parity vector (p_x, p_y, p_y) given as follows: $p_x = x \bmod 2, p_y = y \bmod 2, p_z = z \bmod 2$. The set of parity vectors is called the binary parity code and denoted by $\mathcal{P}(C)$.

Theorem 2 *For a 3-dimensional sensor field with p grid points in each dimension, p even and $p > 2$, target location is achieved with a smallest possible number of sensors ($S_n^p = p^n/4$) if the binary parity code $\mathcal{P}(C)$ is the perfect binary $(3, 1, 3)$ Hamming code, where a perfect (n, k, d) Hamming code consists of 2^k codewords in n dimensions and the minimum distance between codewords is d.*

Theorem 3 *For a three-dimensional sensor field with p grid points ($p > 4$, p even) in each dimension, sensor placement with a minimum number of sensors ($S_n^p = p^3/4$) can be achieved if and only if sensors are placed on grid points whose parity vectors are (0,0,0) and (1,1,1).*

The next theorem addresses cases where p is not necessarily even. For a sensor field with p grid points in each dimension, we can define an n-dimensional p-ary code C with covering radius 2 as follows: C is the smallest set of grid points (vertices) such that each non-codeword is at distance at most two from a codeword.

Theorem 4 *Let $K^p(n, 2)$ be the minimum number of codewords in a p-ary n-dimensional code with covering radius 2. Then for any $p > 4$, an upper bound on the minimum number of sensors S_n^p for target location in an n-dimensional sensor field with p grid points is given by $S_n^p \leq (2n + 1)K^p(n, 2)$.*

Theorem 4 implies that sensor placement can be carried out by first determining a code $K^p(n, 2)$ with covering radius 2. Sensors are then placed on the grid points corresponding to the codewords as well as on all grid points that are adjacent to codewords of $K^p(n, 2)$.

It can also be shown that as the number of grid points in a sensor field tends to infinity, the fraction of sets of targets of cardinality exactly l that are uniquely identifiable approaches one if $l = o(\sqrt{N})$. This underlines the effectiveness of the sensor placement approach for single targets, and implies that separate placement algorithms for multiple targets are not necessary.

References

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