Augmented Terrain-Based Navigation to Enable Persistent Autonomy for Underwater Vehicles

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***Abstract*—To effectively examine ocean processes, sampling campaigns require persistent autonomous underwater vehicles that are able to spend a majority of their deployment time maneuvering and gathering data underwater. Current navigation techniques rely either on high-powered sensors (e.g., Doppler Velocity Loggers) resulting in decreased deployment time, or dead reckoning (compass and IMU) with motion models resulting in poor navigational accuracy due to unbounded sensor drift. Recent work has shown that terrain-based navigation can augment existing navigation methods to bound sensor drift and reduce error in an energy-efficient manner. In this paper, we investigate the augmentation of terrain-based navigation with in situ science data to further increase navigation and localization accuracy. The motivation for this arises from the need for underwater vehicles to navigate within a spatiotemporally dynamic environment and gather data of high scientific value.**

We investigate a method to create a *terrain* map with maximum variability across the range of data available. These data combined with local bathymetry create a terrain that enables underwater vehicles to navigate and localize 1) relative to interesting water properties, and 2) globally based on the terrain and traditional methods. We examine a dataset of bathymetry and multiple science parameters gathered at the ocean surface at Big Fisherman's Cove on Santa Catalina Island and present a weighting for each parameter. We present efficient algorithms to obtain a convex combination of science and bathymetry parameters for unique trajectories generation.

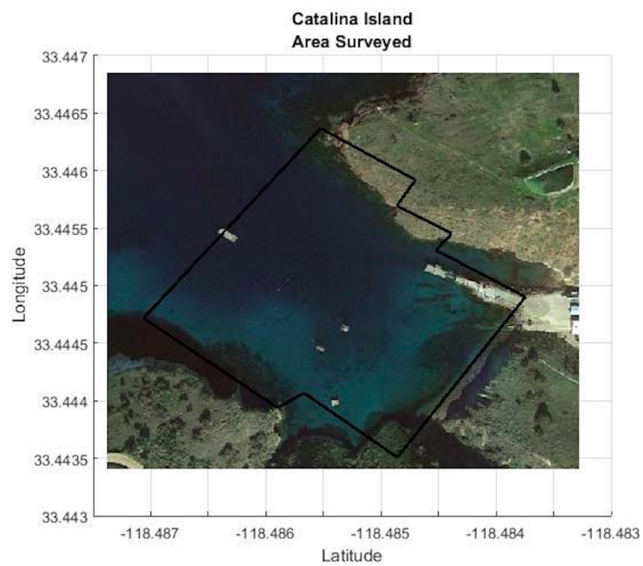


Fig. 1. Surveyed area on Santa Catalina Island.

I. Introduction

Effective study of ocean processes requires sampling over the duration of long (weeks to months) oscillation patterns. Such sampling requires persistent, autonomous underwater vehicles, that have a similarly long deployment duration. The spatiotemporal dynamics of the ocean environment, coupled with limited communication capabilities, make navigation and localization difficult, especially in coastal regions where the majority of interesting phenomena occur. For example, autonomous gliders are a common tool used by ocean scientists to study a range of phenomena in the coastal and deep ocean [1], [2], [3], [4]. Autonomous gliders typically spend 8+ hours underwater, navigating with only a compass, magnetometer and depth sensor. Increasing the surfacing frequency for location fixes/updates limits the amount of data that are collected during a deployment by decreasing the total time underwater, and by expending excess energy for communication and localization while on the surface [5]. Additionally, surfacing in potentially hazardous locations (e.g., shipping lanes) puts the vehicle at risk [6]. Hence, there is a trade-off between navigation accuracy and data collection and safety for the vehicle that must be considered for each mission. Thus, there is a need to increase navigation accuracy while keeping the vehicle underwater as long as possible. Potential solutions with high-powered sensors (e.g., Doppler Velocity Loggers) are feasible, however these also limit the deployment time by utilizing key power resources on-board the vehicle. Here, we approach the problem by using existing sensors and data gathered in situ by augmenting the technique of terrain-based navigation.

Before the advent of satellite based navigation, e.g., GPS, long-distance navigation systems for missiles were developed for long-distance navigation [7]. Cruise missiles needed an accurate, long-term position estimate to guide them to their targets. Basically, data from an embedded altimeter is compared with the ground elevation that is given or predicted by a stored map. Accuracy is dependent on the resolution of the underlying topography map (very good for terrestrial locations) and the accuracy of the measured elevation. This system became redundant after the introduction of GPS, although it is still useful if satellite navigation is unavailable or a satellite connection is lost. Until recently, the utility of terrain-based navigation for underwater vehicles was low due to the poor resolution of bathymetric maps. Updated bathymetry maps beg revisiting the application of this method for low-power, accurate navigation underwater.

This paper examines a combination of bathymetry information and science parameters for creating a terrain map more suitable for localization and navigation of underwater and surface vehicles than a terrain map using only bathymetry information. A map with low spatial auto-correlation and, consequently, high variability is more adequate for a modification of terrain-based navigation for aquatic vehicles.

The proposed methodology has been tested with data from a deployment at Big Fisherman's Cove, at the eastern side of Isthmus Cove on Santa Catalina Island, illustrated in Figure 1, (33°44'N 118°48'W), California, USA, in July 2016.

II. Related Work

A. Terrain-Based Navigation

Terrain based navigation has been around for many decades and was initially designed for use on long-range missiles prior to the development of a robust GPS satellite network [7]. A thorough survey of underwater advances and challenges, with a summary of recent research on Terrain-Based Navigation (TBN) can be found in [8]. A lack of accurate maps is the primary existing shortcoming for TBN for underwater applications. Additionally, limitations in sensors, especially optical range sensors, further limit TBN underwater.

Lagadec presented a simulation of TBN under ice [9], showing the feasibility of using a particle filter for TBN applied to long-term glider navigation. This study was able to utilize low-relief maps (~ 2 km resolution) in the arctic circle to navigate with reasonable accuracy (~ 1000 m resolution). The primary shortcoming of the technique presented in [9] was the lack of a terrain map with appropriate resolution. Lagadec's navigation system also required an accurate motion model of the vehicle, adding to the complexity of the method, and additionally basing a significant portion of the reliability on the accuracy of the ability to estimate acceleration.

B. Dead-Reckoning Navigation

The most popular, energy-efficient method for motion estimation for underwater vehicles is an Inertial Measurement Unit (IMU). IMU error and the associated navigation performance is explored in [10]. Here, the authors show that even using tactile-grade IMU systems, error is still too significant to mitigate all navigation error. Drift and bias propagates through navigation estimates, regardless of the type of IMU employed [10]. Hence, as seen with multiple deployed systems, the use of IMU data must be augmented with additional methods, especially in scenarios when localization fixes are scarce and sensor drift cannot be bounded.

Fig. 2. YSI EcoMapper Autonomous Underwater Vehicle (AUV) during one of the missions.

Based on the results of initial results presented in [11], [12], we are motivated to further investigate the improvement of navigational capabilities of gliders and underwater vehicles by use of TBN and further augmentation. Since most vehicles depend solely upon dead reckoning for subsurface navigation, the uncertainty in the estimated state will grow without bound. For our applications in the coastal regions of Southern California, we generally require the vehicle to surface frequently (every $3 - 6$ hours), see e.g., [11], [13], [14]. Since we acquire GPS ground truth relatively frequently, we are able to bound the growth of the state estimation error. This provides a baseline expected error for the assessment of navigational accuracy and precision. In this paper, we are interested in better localization while underwater between waypoints to enable more accurate reconstructions of executed trajectories.

Here, we propose a method that will augment existing dead-reckoning and TBN methods by incorporating collected science data to mitigate many of these issues that limit existing TBN. Such a method will bound the sensor drift found with IMU navigation. Additionally, with current science sensors able to collect data at rates of 10 Hz and higher, we have the ability to provide high-resolution (~ 1 m resolution) maps for use in navigation and localization. The key innovation of the proposed technique comes from the concept of Environmental or Ecological Niche Models.

C. Ecological Niche Models

Ecological Niche Modeling is derived from one of the primary goals of ecology, which is to map species distribution over geographic ranges and be able to use predictive models to infer where various species are likely to be found [15], [16], [17], [18]. Environmental niche modeling uses a wide range of data to generate a map of a locale showing only chemical and physical parameters that have either been measured or interpolated from direct measurements [19]. Specifically, niche modeling is a method to classify geographic locales as either being habitable or inhabitable by certain species. By monitoring specific physical parameters of an environment and understanding the tolerances of a certain species, it is possible to model where that species will most likely be present [18], [20], [21], [22]. Here, we hypothesize that these niches may also be utilized for underwater vehicle navigation. At this stage, we will assume that the environment is static in both space and time, however the spatiotemporal dynamics of observed ecological niches suggests that they exhibit periodicity or a predictable stochastic behavior, see e.g., [23].

III. Problem Description

In this paper, we examine multiple deployments of a YSI EcoMapper vehicle, illustrated in Figure 2 in Big Fisherman's Cove on Santa Catalina Island, CA USA to develop an augmented TBN methodology to improve navigation and localization. During the deployments, the vehicle navigated on the surface of the water to ground-truth measurements via GPS.

From previous work by the authors, we have a tested and validated TBN methodology [12]. Here, we are interested in augmenting this method with the addition of science parameters. To begin, we assume that depth in not a parameter and approach the problem with the intent to create a *terrain* map (for traditional TBN) from the science parameters available. Hence, the focus of this study is on the approach in determining the appropriate parameters for a given oceanic region along with the respective weights for each of the chosen parameters. The goal of this study is to find a weighting of parameters that achieves the the greatest variability and lowest auto-correlation in space. Upon examining a few scenarios of only science parameters, we additionally investigate the incorporation of depth as a parameter; thus augmenting traditional TBN.

IV. Methods

We investigate methods to create an underlying base map to be used for navigation and localization. The created map is derived from in situ science data collected during an *a priori* mission. To create a map to be used with TBN, we are interested in finding the combination of science data, with associated weights, that provides the *bumpiest* scalar field, i.e., the scalar field with the most likelihood that a given trajectory across it is unique with respect to all others. This corresponds to creating a scalar field with variables that have a low spatial auto-correlation. The proposed methodology was tested with data from multiple deployments at Big Fisherman's Cove. The range of science data that were collected by the vehicle include salinity (ppt), temperature (°C), pH, turbidity (NTU) and concentrations of chlorophyll-a (mg/L), blue-green algae (cells/mL) and dissolved oxygen (mg/L). Figure 3 presents each of the scalar fields for these science parameters on a normalized scale.

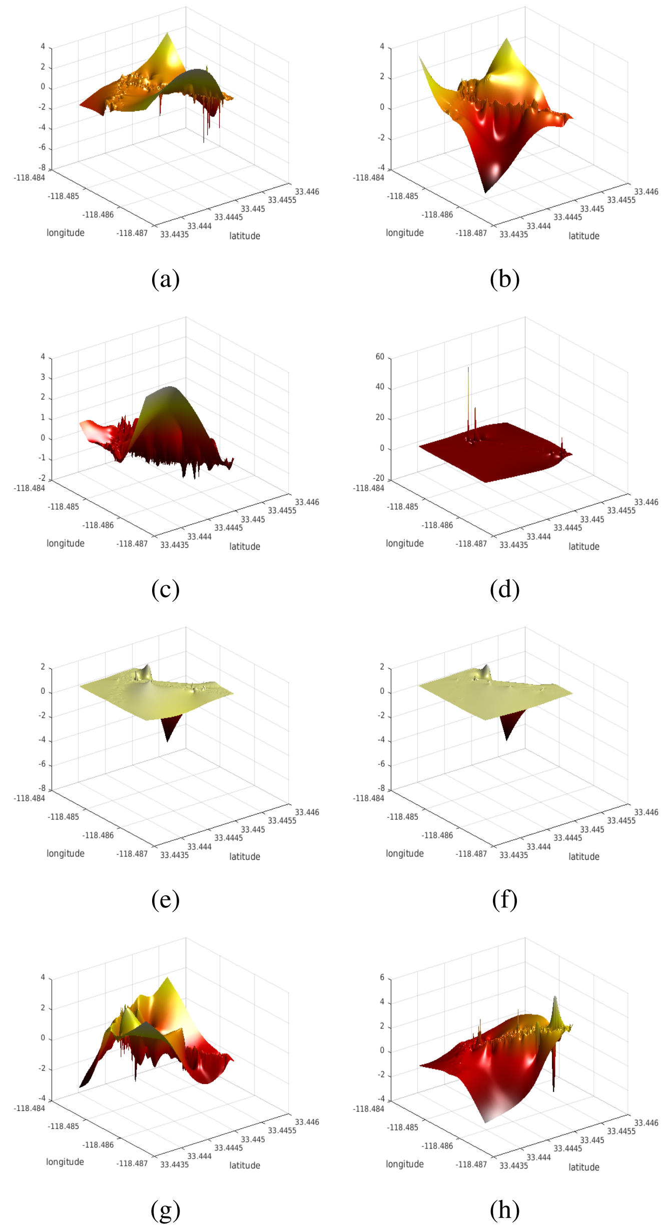


Fig. 3. Normalized scalar fields for (a) salinity; (b) temperature; (c) pH; (d) turbidity; (e) chlorophyll-a; (f) blue-green algae; (g) dissolved oxygen concentration; and (h) depth.

Multiple deployments conducted in Big Fisherman's Cove used a YSI EcoMapper Autonomous Underwater Vehicle (AUV) [24] over a two-day period. The AUV was operated on the surface to ground truth the measurements collected. Using the entire dataset, we generated a base map for each science parameter, as well as depth. The Gridfit function [25] in MATLAB [26] was used for interpolation and extrapolation of the discrete data collected. This method is able to extrapolate beyond the convex hull of the data and builds a scalar field over a given region. The output is a matrix of points for each science parameter. A z-score is a value that indicates how many standard deviations an element is from the mean. A z-score (also known as the standard score value) can be calculated from the following formula:

$$z = \frac{x - \mu}{\sigma} \tag{1}$$

where μ is the mean and σ is the standard deviation of the entire matrix. The global correlation is then calculated for this new matrix of Z-score values. A suitable map for navigation or localization is one such that the spatial auto-correlation is low and a segment of a given trajectory is unique given the map of the entire area of deployment.

A. Global Auto-correlation Score

Our hypothesis is that computing a scalar field with a low auto-correlation will generate a *terrain* map of science data, upon which we can apply a TBN technique. To quantify this, for a science parameter scalar field, we calculate the 2D spatial auto-correlogram, subtract the entry at $(0, 0)$ and sum the absolute values of the entries in this 2D auto-correlogram. This sum is the global correlation. This provides an overall score of autocorrelation for each scalar field. A further assumption of this research is that a combination of science parameters, possibly with bathymetry information, will have greater variability than a single parameter, and even improve the naive TBN approach using only bathymetric data. In this study, we will not compare results of navigation accuracy with other methods, but only seek to develop the technique to create the underlying scalar field. For the interested reader, some results for TBN with only depth can be seen in [27], where the authors achieved desired accuracy for localization of a trajectory traversed by an underwater vehicle.

B. Optimal Map Search

In order to develop the technique to create the underlying scalar field, four different tests were performed seeking the lowest global correlation value:

1) What is the best combination using two different science parameters? What are the coefficients for these variables?
2) What are the coefficients when using all 7 science parameters?
3) Computing global correlation for the bathymetry data.
4) What are the coefficients when using all 7 science parameters combined with the bathymetry data?

In order to address the question in the first test, a global correlation value is calculated for all possible combinations of two variables from the parameters collected. We then find the minimum auto-correlation value [28], which corresponds to the scalar field with most variability. For the second test, a set of coefficients considering the seven science variables is obtained using uniformly random sampling from unit simplex [29]. An array X={$x_1, \ldots, x_{n-1}$} with unique entries from a uniformly random sampling is created with values varying among $\{1, 2, \ldots, M-1\}$ without replacement. The first value is set to 0 and the last value is set to the maximum integer allowed, which is defined as $intmax$. Array X is then sorted in ascendant order. Let Y={$y_1, y_2, \ldots, y_i, \ldots, y_{M-1}$} and y_i is defined as $x_{i+1} - x_i, \forall i \in \{1, 2, \ldots, n\}$. Each entry of array Y is then divided by the sum of all the values of Y so that the new sum is equal to 1. For all variables, let α={$\alpha_1, \alpha_2, \ldots, \alpha_n$} be the set of coefficients for each science data considered for this study. A suitable *terrain* map computed from our proposed approach for navigation and localization is given by the linear combination S presented in Equation. 2

$$\begin{aligned} S =& \alpha_1 * salinity + \alpha_2 * temperature + \\ & \alpha_3 * specific\_conductivity + \alpha_4 * pH + \\ & \alpha_5 * turbidity + \alpha_6 * chlorophyll + \\ & \alpha_7 * blue\_green\_algae + \\ & \alpha_8 * dissolved\_oxygen \end{aligned} \tag{2}$$

Here, S represents a linear combination of the science parameters considered, α is the set of coefficients that minimizes the spatial auto-correlation.

For the third test, only bathymetry information is considered. An augmented TBN has been developed using depth data in [27] and results showed that bathymetry information is a viable approach for creating terrain maps. Here, we examine bathymetry information using the same methods used for addressing questions 1, 2 and 4. Finally, the fourth and last test is addressed by combining the bathymetry infomation with the science parameters as a new approach. Equation 2 is extended to include depth as another variable and analyzing the effect of the bathymetry structure.

V. Results

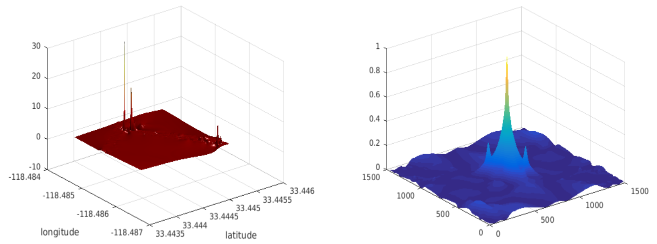
Answering the proposed questions in section IV-B, results for all possible combinations of two science variables show that when salinity and turbidity are combined as follows:

$$2SciVar = 0.38 * salinity + 0.62 * turbidity \tag{3}$$

the global correlation value is 14838.49. This value is minimum compared to all other combinations of two science parameters. The terrain map for this approach and its auto-correlogram in Figure 4. This result indicates the viability of using science parameters for terrain-based navigation since there is a high variability for the terrain map generated.

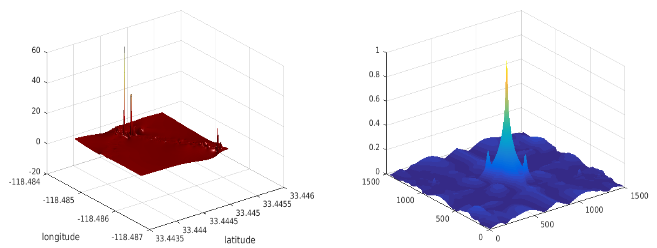
By uniformly picking one million points from a simplex and comparing the global correlation of each combination, results show that the minimum global correlation is 8062.21 after testing for one hundred thousands different set of coefficients. The best global correlation was achieved when:

$\alpha_1 = 0.226828$ (Salinity);



(a) (b)

Fig. 4. For the combination of two science parameters with maximum variability: (a) terrain map; (b) auto-correlogram.



(a) (b)

Fig. 5. For the combination of the seven science parameters with minimum auto-correlation: (a) terrain map; (b) auto-correlogram.

$\alpha_2 = 0.190777$ (Temperature);
$\alpha_3 = 0.096279$ (pH);
$\alpha_4 = 0.450075$ (Turbidity);
$\alpha_5 = 0.00025$ (Chlorophyll);
$\alpha_6 = 0.023924$ (Blue-green algae);
$\alpha_7 = 0.011867$ (Dissolved oxygen).

According to this approach, the turbidity of the water, measured in Nephelometric Turbidity Units (NTU), has the highest coefficient and it is the variable that leads to higher variability of the terrain map, desired for the TBN approach. The terrain map for this approach and its auto-correlogram are illustrated in Figure 5.

In the case of using only the bathymetry information, global correlation is approximately 610096.77. This approach was examined in [27], and results demonstrated an accurate localization of a trajectory traversed by an underwater vehicle when water depth information correlated to local bathymetry maps was used. The auto-auto-correlogram for this case is illustrated in Figure 6. The last and most important results shows that when using a combination of science parameters and the bathymetry data, global correlation value is 3879.67.

Results show that the minimum auto-correlation is achieved when the science parameters are combined with bathymetry information. When only science parameters are used, auto-correlation is also lower than just bathymetry information, justifying the use of science parameters for creating terrain maps for localization and navigation. The combination of science parameters and bathymetry data led to significant reduction in the global correlation when compared to only

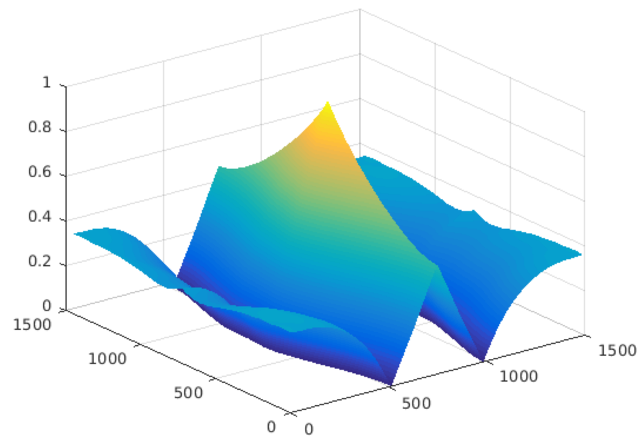


Fig. 6. Auto-correlation for the bathymetry information.

using bathymetry information. This result, in turn, increases the variability and facilitates localization and navigation since any random trajectory extracted from the terrain map will be unique in this body of water. It is known from [12] and [27] that TBN and the use of bathymetry information work well for localization and navigation because the structure of the bathymetry facilitates the unique segment of a trajectory to be found. When combining more science with bathymetry information, the generated scalar fields terrain maps are optimized for a TBN.

Table I shows the global correlation values for the best variability achieved for a combination of two science parameters; best combinations of the seven science parameters; only the bathymetry information and the combination of the seven science parameters and the bathymetry information.

TABLE I
GLOBAL CORRELATION VALUES FOR DIFFERENT COMBINATION OF PARAMETERS.

Parameters	Global correlation
Salinity and Turbidity	14838.49
Science combined	8062.21
Bathymetry	610096.77
Science and bathymetry	3879.67

VI. CONCLUSIONS

A map constructed using in situ science data in combination with bathymetry was developed for improved navigation and localization accuracy for aquatic vehicles. The incorporation of science data increased the global correlation leading to greater variability and a more suitable map for localization and navigation using an augmented TBN. The methods presented in this paper can serve as an important technique to create a *terrain* map with maximum variability across the range of data available.

However, this research examined only one deployment in a coastal ocean region and the parameters associated with this

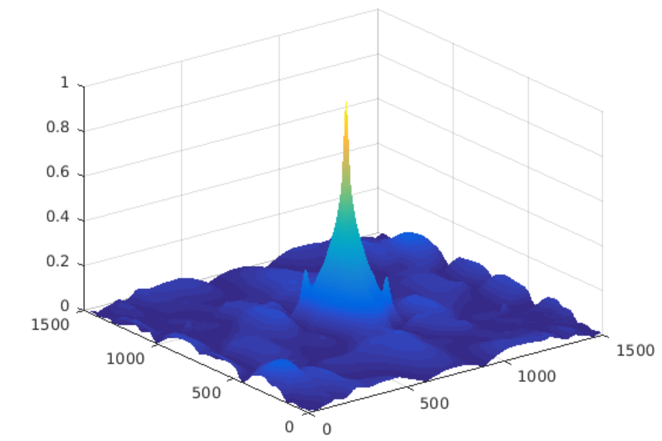


Fig. 7. Auto-correlation for the combination of science parameters and bathymetry information.

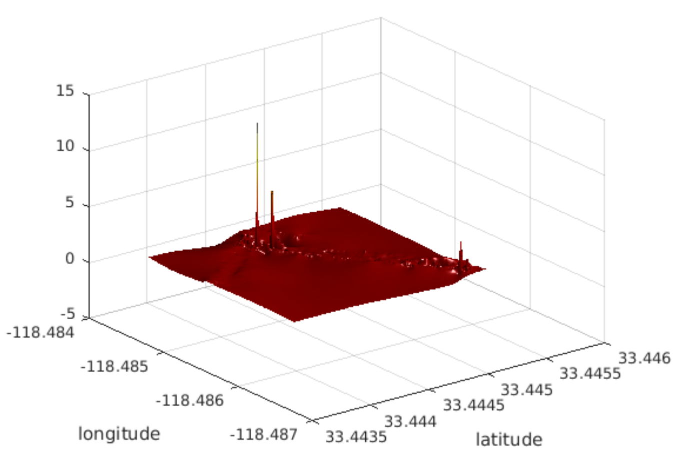


Fig. 8. Terrain map generated for the combination of science parameters and bathymetry information.

location will be unique to this region. Therefore, any random trajectory extracted from the terrain map will be unique to that area. This is what makes localization possible though an augmented TBN. When satellite navigation is unavailable, as is the case underwater, an augmented TBN with bathymetry and science information may be a promising method for localization and navigation. The utility of TBN for underwater vehicles became valuable with the increase of resolution of bathymetric maps, and the proposed method further refined these maps with the supplementation of more data. Furthermore, the incorporation of science parameter may lead to a low-power and accurate navigation technique for underwater vehicles.

VII. Future Work

The map generated with the combination of science parameters and bathymetry information can facilitate localization and navigation algorithms for underwater and surface vehicles with a vanilla application of traditional TBN methods. For most ocean science applications, there is a need for underwater vehicles to navigate within a spatiotemporally dynamic environments and to gather data of high scientific value. Thus, it will be of interest to investigate methods that are able to adequately propagate critical *ecological niches* in a spatiotemporal fashion to maintain the reliability upon then for navigation or relative localization. Here, instead of thinking of locations existing in geographic space, we consider them to be drawn from or existing in an environmental space. Coupled with physical models (predictive ocean models), this relaxes the dependence on geographic coordinates for navigation, and enables the deign of methods for improving navigation and sampling within a dynamic feature. The inclusion of depth as a parameter does serve to ground-truth this methodology as we continue to develop the supporting architecture for spatiotemporal dynamics.

We have determined the unique parameters and their associated weights to augment TBN for a certain coastal ocean setting. Future work will include examining other bodies of water in other locations in the ocean to see if similar results for a unique set of parameters can be determined. This is an exploratory paper and promising results were found when examining the on Santa Catalina Island. Different deployments should still be examined taking in consideration the spatiotemporal dynamics of the science parameters.

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