Coming to Our Senses: From Constructivism to Democratization of Math Education

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Motivation: Paralleling my own transformation from a Platonist to a radical constructivist, mathematics education has been experiencing for more than a decade a movement that started in theoretical foundations mostly originating in von Glasersfeld’s work, and then reached professional organizations, which have been leading extensive efforts to reform school mathematics according to constructivist principles. However, the theories espoused by the researchers are, as yet, too abstract to lend themselves readily to implementation in the classroom. Purpose: I define a shared experiential language (SEL) for the constructivist teacher to embody in order to transform her practice congruently according to constructivist principles. While SEL is comprised of Neuro-Linguistic Programming (NLP) subjective experience distinctions, what “makes it tick” is the constructivist epistemology with its insight that for consistent understanding to happen, new knowledge has to attach to prior experiences in a process of co-construction. Throughout the paper, I elaborate and validate this insight by numerous examples. Practical implications: utilizing SEL allows understanding of mathematics to be rooted in each student’s individual sensory experiences, thus shifting the responsibility for success in mathematics from the students back to those who guide them in co-constructing knowledge. This, in turn, should allow everybody access to understanding and so it should no longer be socially acceptable to fail in mathematics. Key words: Radical constructivism, math education, Neuro-Linguistic Programming, sensory experience, behavioral cues, democratization.

The child cannot conceive of tasks, the way to solve them and the solutions in terms other than those that are available at the particular moment in his or her conceptual development. The child must make meaning of the task and try to construct a solution by using material she already has. That material cannot be anything but the conceptual building blocks and operations that the child has assembled in his or her own prior experience. — Glasersfeld (1987, p. 12)

Introduction

Having been trained in the Platonism of traditional mathematics, my first “Learning III” experience that Bateson defined as one in which “there is a profound reorganization of character” (Bateson 1972, p. 301) — occurred in the late 80s when I started studying Neuro-Linguistic Programming (NLP). NLP is a set of models of subjective experience created for the purpose of making explicit and emulating in oneself and others strategies of excellence (Dilts et al. 1980). Its primary tenet (formulated originally by A. Korzybski) is “The map is not the territory,” which, in the words of Watzlawick (1984), means that “the name is not what it names; an interpretation of reality is only an interpretation and not reality itself. Only a schizophrenic eats the menu instead of the foods listed on the menu” (p. 215). I embraced this tenet, and as a consequence a shift in my view occurred that turned my life around: I moved from the modernist’s belief in an objective reality accessible by reason and observation to the postmodernist’s belief in subjectivity (Pasztor & Slater 2000). Having grown up in a communist country, Watzlawick’s (1984) words struck a chord with me: any system that denies that it operates on a map of reality, rather than on reality itself, will not only be unable to recognize and adjust to changes in its perception of reality, but will also be unable to tolerate any other representation of reality. I have had first hand experience of examples that “go from the ridiculous to the gruesome” of a totalitarian regime’s “paradoxical, recursive logic” that typically characterizes paranoia: “It is inherent to the concept of paranoia that it rests on a fundamental assumption that is held to be absolutely true. Because this fundamental assumption is axiomatic, it cannot and need not demonstrate its own veracity. Strict logical deductions are then made from this fundamental premise and create a reality in which any failures and inconsistencies of the system are attributed to the deduction, but never to the original premise itself” (ibid., pp. 223–224). Whoever criticizes the premises of the system is therefore declared to be an enemy and will not be tolerated.

Ten years later, my then therapist and now co-author and friend, Mary Hale-Haniff, introduced me to constructivist therapies. What a shake-up I had when I read in Lynn Hoffman’s (1990) paper an account of Heinz von Foerster criticizing NLP’s tenet, “The map is not the territory,” and confronting it with his own view that “The map is the territory”? Once again, I embarked on a “Learning III” experience, and it all fell into place when I read von Glasersfeld’s (1984) introduction to radical constructivism. It clicked. It fits perfectly with most aspects of my life — some conscious, some unconscious. It fits with my dissatisfaction with the hierarchical teacher-student, physician-patient, therapist-clients and other similar relationships, and my deep distrust of statistics and other quantitative research methodologies. I came to understand that NLP’s epistemology was incongruent with its overall intents. Its map-territory
The traditional approach to mathematics education

The traditional, positivist approach to instruction has been referred to as "the Age of the Sage on the Stage" (Davis & Maher 1997, p. 93), due to its "transmission" model of teaching, where teaching means "getting knowledge into the heads" of the students (Glaserfeld 1987, p. 3), that is, transmitting knowledge from the teacher to the student. The underlying philosophy is that knowledge is out there, independent of the knower, ready to be discovered and be transferred into people's heads. It is "a commodity that can be communicated" (Glaserfeld 1987, p. 6). The ontology presupposed in this view is that there is one true reality out there, which exists independently of the observer. Furthermore, we have access to this reality, and we can fragment, study, predict and control it (Lincoln & Guba 1985; Hale-Haniff & Pasztor 1999).

However, as von Glaserfeld (1987, p. 4) points out, while trying to access reality, we have been caught in an age-old dilemma: If truth is defined as "the perfect match, the flawless representation" of reality, who is to judge "the perfect match with reality"?

To answer this question, Western philosophy has taken a route in which, given the right tools, pure reason is believed to be able to transcend all social and cultural constraints and the confines of the human body, including those of perception and emotion. Mathematical reasoning has been seen as the purest example of reason: "purely abstract, transcendental, culture-free, unemotional, universal, decontextualized, disembodied, and hence formal." (Lakoff & Nuñez 1997, p. 22; for more "finely tuned" control see Lakatos 1976). The traditional scientist, mathematician, or, in general, researcher, is out to find objective truth. In doing so, she is trained to be value-neutral in order to be able to objectively "the perfect match" with reality. She is a "cool, detached, solitary genius, the one who has the answers that others don't have, as if the truth could be owned" (Pert 1997, p. 315).

In practice, however, there is a direct "relationship between claims to truth and the distribution of power in society" (Gergen 1991, p. 95). Those at the top of the educational system hierarchy are the "objective" experts of knowledge; they determine teaching goals and criteria of assessment. Accordingly, the traditional teacher–student relationship is a hierarchical, authoritarian relationship.

The constructivist view of knowledge and its implications for mathematics education

In contrast to positivist philosophy, constructivist philosophies have adopted a concept of knowledge that is not based on any belief in an accessible objective reality. In the radical constructivist view, knowing is not matching reality, but rather finding a fit with observations. Constructivist knowledge is "knowledge that human reason derives from experience. As such it has an all-important function: It enables us to solve experiential problems" (Glaserfeld 1987, p. 5). With this theory of knowledge, the experiencing human turns "from an explorer who is condemned to seek 'structural properties' of an inaccessible reality… into a builder of cognitive structures intended to solve such problems as the organism perceives or conceives" (ibid.).

Now, let us look at the two views that are so often confused with the tenets of radical constructivism (Pasztor 2004a): 1. there exists a mind-independent reality (MIR), albeit only indirectly accessible, and 2. there exists no MIR. The first view is close to the positivist ontology, except now we do not have the possibility of a "perfect match," but only that of a mediated match. Still, who is going to judge the "better" match?

A constructivist view is inconsistent with both of these ontological views. As von Glaserfeld (2004a, 2) states, the constructivist holds that "all coordination and, therefore, all structure is of the organism's own making," and therefore he has no way of knowing anything about the ontological reality of these constructs. In fact, he has no way of knowing anything about an MIR. Furthermore, as soon as we posit the existence or non-existence of an MIR, we have caused a split between the knower and the known. The one
who knows whether an MIR exists or does not exist becomes the expert, the authority. In the constructivist view, a third person has no way of knowing anything about my or your own experience. As von Glaserfeld (2004b, [4]) says, “someone else is always my construction.” The only expert of your experience is you. This view, as I will show, can make a tremendous difference in math education.

For more than a decade now, mathematics education in the US has been experiencing a top-down reform movement that started with the theoretical foundations of mathematics education that mostly originated in von Glaserfeld’s work, and then moved to the professional organizations, which then started and have since been leading extensive efforts to reform school mathematics according to constructivist principles (NCTM 2000). So far, however, the reform has been moving only very slowly into the mathematics classroom practices. Besides complex political reasons (Alacaci & Pasztor 2002), one of the reasons for this is that the theories espoused by the researchers to implement constructivist principles are, as yet, too abstract to readily lend themselves to implementation. One of the goals of my own research efforts in math education has been to help translate the language of these theories into the experiential language of students.

Abstract mathematical concepts are metaphorical and are built from people’s sensory experiences (Lakoff & Núñez 1997; Lakoff & Johnson 1999). The constructivist teacher’s role is to make sure that they fit the students’ individual experience. Frustation and confusion ensue if the teacher’s metaphorical mapping is rooted in an a-priori construction, rather than in the student’s own experience. English (1997) provides a very good example of what happens in such a case. It concerns the use of a line metaphor to represent our number system, whereby numbers are considered as points on a line. The “number line” is used to convey the notion of positive and negative numbers, and to visualize relationships between numbers. It turns out that students frequently have difficulty in abstracting mathematical ideas that are linked to the number line (Dufour-Janvier, Bednarz & Belanger 1987, quoted in English 1997, p. 8).

“No tendency for students to see the number line as a series of ‘stepping stones,’ with each step conceived of as a rock with a hole between each two successive rocks. This may explain why so many students say that there are no numbers, or at the most, one, between two whole numbers.”

While students are often able to reorganize their experience in a way that makes it fit the constraints of the problem at hand, often times the teacher needs to provide for the students “precisely those experiences that will be most useful for further development or revision of the mental structures that are being built” (Davis & Maher 1997, p. 94). This idea is wonderfully demonstrated by Machtinger (1965) (quoted in Davis & Maher, 1997, pp. 94–95) who taught kindergarten kids to conjecture and prove several theorems about numbers, including even + even = even, even + odd = odd, and odd + odd = even. She did so by defining a number n as “even” if a group of n children could be organized into pairs for walking along the corridor and as “odd” if such a group had one child left over when organized into pairs. Since walking along the corridor in pairs was a daily experience for the kids, learning the new information became a matter of just expanding or reorganizing their existing knowledge.

But this is not always possible. In particular it is not possible when the teacher uses incommutable metaphors to explain mathematical ideas. I was shocked and saddened by the great regret with which the 86-year-old Carl Jung remembered in his 1962 memoirs the terror that he experienced in math class. While his teacher gave the impression that algebra was very natural, the young Jung failed to understand what numbers actually were. He knew they were not flowers, nor animals, nor fossils – they were nothing he could imagine. They were just amounts that resulted from counting. To his greatest confusion, these amounts were replaced by letters the meaning of which was a sound. His teacher tried hard to explain the purpose of this strange operation of replacing understandable amounts by sounds, but to no avail. This, what seemed to Jung to be a random expression of numbers through sounds such as “a,” “b,” “c,” or “x,” did not explain anything about the nature of numbers. His frustration peaked with the axiom, “if a = b and b = c, then a = c,” since by definition it was clear that “a” denoted something different from “b,” and so could not be equalled with “b,” let alone with “c.” He was outraged. An equality could be “a = a,” but “a = b” was a lie and deceit. His intellectual morality resisted such incongruities that blocked his access to the understanding of mathematics. To his old age Jung had the uncorrectable feeling that if he could have accepted the possibility of “a = b,” that is, of “sun = moon, dog = cat, etc.,” then mathematics would have infinitely absorbed him. Instead, he came to doubt the morality of mathematics for his entire life. Like so many others, he came to doubt his own self-worth, which, back then, prevented him from asking questions in class (Jung 1962).

In practice, “[f]or too many people, mathematics stopped making sense somehow along the way. Either slowly or dramatically, they gave up on the field as hopelessly baffling and difficult, and they grew up to be adults who – confident that others share their experience – nonchalantly announce, ‘Math was just not for me’ or ‘I was never good at it.’” (Askey 1999). Ruth McNeill shares her story of how she came to quit math: “What did me in was the idea that a negative number times a negative number comes out to a positive number. This seemed (and still seems) inherently unlikely – counterintuitive, as mathematicians say. 1 … could not overcome my strong sense that multiplying intensifies something, and thus two negative numbers multiplied together should properly produce a very negative result” (McNeill 1988, quoted in Askey 1999).

Most mathematical concepts being metaphorical and understanding a metaphor meaning successfully mapping concepts from our individual experience onto new domains, teaching the metaphorical structure of mathematics becomes indispensable. It shifts the definition of “mathematical understanding” from a goal that only a few “talented” or “gifted” people can reach, to a process rooted in all people’s individual experience.

Is $2 + 2$ still 4?

If objectivity of mathematics is just a myth, what happens to basic facts such as $2 + 2 = 4$? Are we denying them? The question is very nicely answered in a dialogue between von Foerster and von Glaserfeld in their (1999) book. The following is an excerpt from the book (translated from German by myself).
the rules, we come to the result determined by these rules. And with the action of counting the directions plays no role, but according to the rules, we may count each unit only once. This is the number constancy" (Foerster & Glaserfeld 1999, pp. 133–134).

So, while mathematics is a human construction, it is not an arbitrary creation. It is “not a mere historically contingent social construction. What makes mathematics non-arbitrary is that it uses the basic conceptual mechanisms of the embodied human mind…. Mathematics is a product of the neural capacities of our brains, the nature of our bodies, our evolution, our environment, and our long social and cultural history’’ (Lakoff & Nuñez 2000, p. 9).

Operative learning and learning states

In constructivism, the meaning of learning has shifted from the student’s “correct” replication of what the teacher does to “the student’s conscious understanding of what he or she is doing and why it is being done” (Glaserfeld 1987, p. 12). “Mathematical knowledge cannot be reduced to a stock of retrievable ‘facts’ but concerns the ability to compute new results. To use Piaget’s terms, it is operative rather than figurative. It is the product of reflection – and whereas reflection as such is not observable, its product may be inferred from observable responses” (Glaserfeld 1987, p. 10). Operative knowledge is constructive. “It is not the particular response that matters but the way in which it was arrived at” (Glaserfeld 1987, p. 11).

But how is the student to attain such operative knowledge in mathematics, when the “structure of mathematical concepts is still largely obscure” (Glaserfeld 1987, p. 13)? Most definitions in mathematics are formal rather than conceptual. In mathematics, definitions merely substitute other signs or symbols for the definendum. Rarely, if ever, is there a hint, let alone an indication, of what one must do in order to build up the conceptual structures that are to be associated with the symbols. Yet, that is of course what a student has to find out if he or she is to acquire a new concept” (Glaserfeld 1987, p. 14).

To illustrate this point, let us look at an example. While talking about my research to J, a doctoral student in Computer Science in his mid thirties, I asked him to solve a word problem. “Word problem? I hate word problems!” was J’s response even before he knew what the word problem was. The word problem was this: "Joey has a new puppy. His sister, Jenna, has a big dog. Jenna’s dog weighs eight times as much as the puppy. Both pets together weigh 54 pounds. How much does Joey’s puppy weigh?" I listened to the problem, and then asked him to repeat it. As I did so, I made the following notes, turning his back to me:

- puppy: x
- big dog: 8x
- x + 8x = 54
- 9x = 54

Then he stopped and said he didn’t know his multiplication table. “So anyway, what is the answer?” I asked. J blushed and became restless. “What do you mean?” he asked. I replied, “Well, what was the question?” After Jeff repeated the problem’s question, I asked again, “So, how much does the puppy weigh?” Again, J didn’t answer but became instead more and more insecure. “Why, did I do something wrong? I must have screwed up somewhere.” “No,” I replied. “All I have in mind is how do you get that x?”

J was so fixed on getting the exact number as a result, that it never occurred to him to say something like “The puppy weighs 54 divided by 9, whatever that is.” Instead, he questioned his whole approach thinking he had “screwed up somewhere.” I asked him why he hated word problems. He replied, “Because they make me feel stupid.” How? I inquired. “Well, if I don’t get an immediate answer, I feel stupid. It is stuff I should know. It is expected of me.” Jeff went on to talk about the time when he came to hate word problems. He never understood what the teacher did in class – he failed to see any pattern in these word problems. The teacher had them solve word problems either under time pressure or at the board, in front of the entire class. He felt threatened and never actually got over it.

There is a general agreement across the constructivist research in mathematics education that for consistent understanding to happen, new knowledge has to attach to students’ prior experiences. But just what kind of prior experiences? Which ones are optimal for new learning? How can a teacher behave in a way as to resurrect those experiences? What
Many people argue that they do not think in images, but rather in words or abstract symbols. But "most of the words we use in our inner speech, before speaking or writing a sentence, exist as auditory or visual images in our consciousness. If they did not become images, however fleetingly, they would not be anything we could know" (Damasio 1994, p. 106).

Damasio (1994) goes as far as to require as an essential condition for having a mind the ability to form internal (visual, auditory, kinesthetic) images, and to order them in the process we call thought. His view is that "having a mind means that an organism forms neural representations which can become images, be manipulated in a process called thought, and eventually influence behavior by helping predict the future, plan accordingly, and choose the next action" (p. 90).

Sensory images are often referred to as "mental representations" – a term that, as von Glasersfeld (1987) explains, can be quite misleading: "In the constructivist view, 'concepts,' 'mental representation,' 'memories,' 'images,' and so on, must not be thought of as static but always as dynamic; that is to say, they are not conceived as postcards that can be retrieved from some file, but rather as relatively self-contained programs or production routines that can be called up and run (cf. Damasio's 1994 dispositional representations). Conceptions, then, are produced internally. They are replayed, shelved, or discarded according to their usefulness and applicability in experiential contexts. The more often they turn out to be viable, the more solid and reliable they seem. But no amount of usefulness or reliability can alter their internal, conceptual origin. They are not replicas of external originals, simply because no cognitive organism can have access to 'things-in-themselves' and thus there are no models to be copied" (p. 219).

How constructivism honors other ways of knowing and communicating

Positivist methodology privileges auditory-verbal communication, often to the exclusion of other modalities. Thus we teach the verbally oriented conscious mind, and often ignore visual and kinesthetic aspects of experience. However, if we intend to communicate in a holistic manner engaging all of our senses, we need to also honor other ways of knowing.

"For the constructivist teacher – much like the psychoanalyst – 'telling' is usually not an effective tool. In this role, the teacher is much less a lecturer, and much more of a coach (as in learning tennis, or in learning to play the piano). A recent slogan describes this by saying 'the Sage on the Stage has been replaced by the Guide on the Side.' It is the student who is doing the work of building or revising [...] his or her] personal representations. The student builds up the ideas in his or her own head, and the teacher has at best a limited role in shaping the student’s personal mental representations. The experiences that the teacher provides are grist to the mill, but the student is the Miller" (Davis & Maher 1997, p. 94).

The holistic, constructivist view presupposes that the teacher should have the potential to attend to all aspects of sensory experience and communication both in herself and in the student’s system. In addition to auditory-verbal aspects, visual and kinesthetic experience may also be privileged, with both unconscious (tactile) and conscious communication and perception considered. When teachers are (implicitly) trained to ignore communications related to intra-personal, emotional, and unconscious experience, we are imparting positivist principles. Most of us have been socialized largely according to positivist thinking, conceptualizing emotions as sudden and intense experiences that come and go at certain times; something that a sane or balanced person learns to keep under control so that rational thinking and control can prevail. On the other hand, the holistic, constructivist view depicts emotional experience as ongoing, simultaneous with and supportive of, the rest of experience.

Kinesthetic experience is ever-present (although not always consciously accessible) in form of “body images.” By dint of juxtaposition, body images give to other images a quality of goodness or badness, of pleasure or pain. I see feelings as having a truly privileged status… [F]eelings have a say on how the rest of the brain and cognition go about their business” (Damasio 1994, pp. 159–160).

It is important to note that experience that is kinesthetic to one person (e.g., the student) is accessible primarily visually to the other (e.g., the teacher). For example, as the student...
feels or his or her face get hot, the teacher might notice him blush. Or, as the student feels a sense of pride welling up in him, the teacher might notice him taking a deep breath as he squares his shoulders. Thus learning to detect new categories of sensory experience in oneself and others involves enhancing perception of new categories of both kinesthetic and visual experience. Becoming more consciously aware of categories of sensory experience other than auditory-verbal, the teacher enhances her ability to accommodate to the students’ experiences.

Submodalities: Refining the see/hear/feel building blocks

Each sensory modality is designed to 'perceive' certain basic qualities called submodalities, of the experience it represents (Bandler & MacDonald 1988; Passtor 1998; Hale-Haniff & Passtor 1999). Visual submodalities refer to qualities such as: location in space, relative size, hues of color or black and white, presence or absence of movement, rhythm, degree of illumination, degree of clarity or focus, flat or three-dimensional; associated or dissociated (seeing oneself in the image, or viewing from a fully associated position). Auditory submodalities refer to qualities such as location, rhythm, relative pitch, relative volume, content: voice, music, noise. Kinesthetic submodalities include such qualities as: location of sensations, presence or absence of movement (and if moving, the physical locations of sequential sensations), the type of sensations: temperature, pressure, density, duration, moisture, pervasiveness of body area involved, sense of movement and accleration, changes in direction and rotation.

Submodalities are distinctions that separate experiences from one another. As such, their significance comes to bear only when we contrast submodalities of images that represent different experiences. To illustrate this, let us look at the submodalities of different experiences of my husband, specifically at how different contexts are manifested in completely different sets of submodalities. My husband is an architect and he is quite proficient in geometry. First, here is what he reports regarding his experience of abstractation: "As part of a math problem involving triangles, an abstract triangle occurs first as a fuzzy shape without any material body. It doesn’t have a surface; not even a clear boundary. Its size is also changing between a couple of inches to one or two feet. It is quite far from my face and its distance is unspecified but it is still in the room. As a consequence, its shape, size, and location can easily be manipulated. As it is manipulated, such as made equilateral or rotated, these parameters change rapidly. The boundary becomes more defined, the size concrete, and the distance fixed. It still remains, however, a line-drawing without a body or surface. It is always a colorless figure, either gray or black and white." In contrast, for my husband imagining an emergency triangle on the road propped up behind a car "is a vivid picture with concrete shape, thickness, material, etc. It is red with white edges in fluorescent colors set against the gray asphalt background. I see it at a distance of 10 feet in life size, that is, the same size I would probably see it driving by and looking at it from this same distance. I feel some anxiety in my stomach as I probably connect this picture unconsciously with a car break-down or an accident."

Sensory strategies: sequences of see/hear/feel blocks leading to a particular outcome

Our thought processes are organized in sequences of images that have become consolidated into functional units of behavior leading to a particular outcome and often executed below the threshold of consciousness. They are called sensory strategies (Dilts et al. 1980) Each image triggers another image or a sequence of images. For example, you hear X’s name, this triggers your remembering X’s face, close up, somewhat distorted, and pinkish red, which, in turn, triggers a negative feeling. Over time, each image or sequence of images comes to serve as a stimulus that automatically triggers other portions of the perceptual or recalled experience it represents. The creation of such triggers happens through learning and depends on various complex subjective, social, cultural and other factors. I will illustrate the idea of sensory strategy with a few examples from a pilot project I conducted in the academic year 1999–2000 with a class of fourth graders with the aim of teaching them SEL and through it, awareness of their mental processes while solving math problems.

Ramon chose the following problem to solve: Which measure is the best estimate to describe the length of the salamander below (picture followed text)? Circle the best estimate: 3 inches 3 miles 3 pounds.

Here is what he reported: "What I did was picture a huge ruler in front of my face and I saw the numbers 1,2,3,4,5,… I looked at the picture [in the book] and compared it with 3 inch and it was right. Besides, pounds is weight and miles is larger than inch."

Kevin’s strategy for implementing a pattern is also quite remarkable. I asked the class to multiply 1 x 1 (= 1), 11 x 11 (= 121), 111 x 111 (= 12321), and 1111 x 1111 (= 1234321).

Then I asked them to continue the pattern. Kevin reported the following for calculating 11111 x 11111: "First I looked, then [knocking with his left hand on his head just above his left ear] I heard ‘tap, tap-tap, tap-tap-tap, tap-tap-tap-tap, tap-tap-tap-tap, tap-tap-tap-tap,’ and then back down tap-tap-tap-tap, tap-tap-tap, tap-tap, tap.” He followed this by writing 123454321.

We each have our strategies in terms of what we see, hear, or feel, of getting out of bed in the morning, multiplying two numbers, deciding when to buy gas, or knowing that something is right. For example, Melanie in my pilot project repeatedly demonstrated a distinct problem solving strategy that lets her know that the result is “right.” Let us look, for example, how she solved the following multiple choice problem: Alana entered the county spelling bee. She spelled 47 words correctly before she made a mistake. If she had spelled three more words correctly, she would have spelled twice as many words as last year. How many words did she spell correctly last year?

Here is how Melanie explained her solution (in terms of what she saw, heard or felt) in her homework: "I added each number to itself and 25 + 50 = 75. The problem says 47 then + 3 = 50. I did not feel anything but in my head I saw 47 + 3 = 50. I also saw that 50 was really gold and yellow and it was blinking and heard it beep. Bleep, beep, beep, beep it sounded really fast and loud. My head was
By also attending to process rather than only to content, the teacher can detect order or pattern, using other ways of knowing besides rational logic, such as attending to physiological and language cues.

Although sensory experience is simultaneously available to all senses, people attend to various aspects of see-hear-feel experience at different times, which is manifested in their language. For example, let us take the case of two children trying to work together on a mathematics problem. One child does “not see” what they are supposed to do, while the other states she doesn’t get “a feel” for what they are supposed to do. In this scenario, communication flow is obstructed because each child is attending to a different sense system, or logical level of experience (Bateson 1972). By noticing this, the teacher can help the children translate their experience so it can be shared and attention can again flow freely. Sensory system mismatches often take place between teachers and children. For example, if a child says, “Your explanation is somewhat foggy,” the teacher’s response of matching the visual system by asking “What would it take to make it clearer?” might be a better fit than the kinesthetic mismatch of “So you feel confused?”

People’s sensory strategies are processes that cause “changes in body state – those in skin color, body posture, and facial expression, for instance – [which] are actually perceptible to and external observer.” (Damasio 1994, p. 139). These physical reactions are important cues for the external observation and confirmation of people’s sensory strategies. The primary behavioral elements involved are: language patterns, body posture, accessing cues, gestures, and eye movements (Dilts et al. 1980; Pasztor 1998; Hale-Haniff & Pasztor 1999).

Attending to the sense system presupposed in people’s language is based on the assumption, derived from constructivist therapy case studies and literature, that sensory experience or “the report of the senses” reflects the interaction between body and mind, and that one can attend to communication behavior as a simultaneous manifestation of sensory experience. For example, constructivist therapies are particularly successful in using linguistic metaphors such as “That’s a murky argument,” “Things were blown out of proportion” or “Shrink the problem down to size” (visual); “This is an unheard of solution,” “It has a nice ring to it” or “He talks in circles” (auditory); and “It feels right.” “The solution hit me” or “This is hot stuff” (kinesthetic), as an expression of people’s sensory experiences (Bandler & Mac- Donald 1988; Pasztor 2004b).

Most often, we do not need training to understand the language of behavioral cues. For example, if a person is using gross body movements – large motor movements compared to fine motor movements – we instinctively know what the relationship between level of detail and abstraction in the submodalities of his internal processing is. It would be really odd for that person to say, “I got the details, now give me the big picture.” The more precise the body language, the more precise the “chunk size” of the information the person is processing. We can also tell the high degree of detail by the narrowing of the gaze – it’s almost as if the person was focusing on a particular area of the fine print as opposed to a diffused thing, like noticing a page or a computer screen. Duration and intensity of gaze, coordination of eye and head movements, head tilt and angle, chin orientation (up, down and middle) – some of these are access-
cuing. They might tell us the state that people are in, the configuration of their attention, level of detail, what they are attending to.

Sometimes people lean their head to one side when they are receiving new information, and to another side when it is “a rerun.” Noticing these cues can be very helpful to see that the person is receptive to what we’re saying or when their system is closing down a bit. In the latter case, how can we shift the way we are presenting information so that they open back up again?

Let us say a person wanted to learn the subject area and we noticed their physiology starting to shut out new information. Being able to map the precise point where they shut down and to figure out what was going on that caused them shut down can be helpful to facilitate their getting back in state.

Awareness of behavioral cues also has the benefit of dispelling misconceptions that parents and teachers often have about the children’s behavior. You have probably heard parents or teachers say to their children, “The answer is not on the ceiling!” while forcing them to look down on their notebooks when doing their homework or taking a test. In fact,
Democratization of math education: Utilizing SEL

The premise for utilizing SEL is that if the teacher embodies the distinctions of SEL, I was able to understand that the kids who had said, “All,” had imagined a thinner paint that got underneath the cube and into the cracks between the unit cubes, while the ones who felt confused, imagined the paint “too” thick and concluded that it my not cover the cube evenly enough to have whole unit cubes covered. Ultimately, I was able to separate students’ images of the paint from mine, and thus realize that I had actually specified the problem poorly.

The key to utilizing SEL is to model students’ subjective experience to help them amplify successful learning states by bringing them into consciousness, and, if necessary, to help them shift un-resourceful learning states so that they become resourceful. The premise is that experiences are like a series of dominoes: the more dominoes are falling, the more difficult it is to break un-useful learning patterns. If we can find the first domino or what has knocked down the first domino, so to speak, then the person has much more choice than when his negative response – be it anger, frustration, or helplessness – is real high. It is much more likely that a student has choice while his response to a negative state of learning is still small, and it gives him a sense of control to be able to change it. Through the process of modeling students’ experiences we can slow down their processing so they are able to gain conscious control over their sensory strategies and thus gain conscious mathematical competence.

By rooting mathematics understanding in each student’s individual sensory experiences, we are shifting the responsibility for success in mathematics from the students back to those who guide and lead the process of co-constructing knowledge. This, in turn, should radically change prevailing beliefs about who should be studying mathematics and who should be successful at it: everybody has access to understanding, not just those who possess the “math gene” – it should not be socially acceptable anymore to fail in mathematics.

Acknowledgements

I gratefully acknowledge the support of the National Science Foundation grant CNS-0454211 with the CATE Center at Florida International University. I would also like to thank Mary Hale-Haniff for having accompanied and often guided me on my voyage through paradigms.
References


Received: 24 August 2006

Accepted: 2 December 2006

2007, vol. 2, nos. 2–3