COT 3420
Section U01
Spring 2012

EXAM # 1 ANSWERS

QUESTIONS

Question 1. (30 points)
1. a 2. c 3. a 4. a 5. d 6. b 7. b 8. b 9. b 10. c 11. d 12. d 13. a 14. c 15. a

Grading Criteria: 2 points for each correct answer.

Question 2 (20 points)

Basis: $n = 0$. Then $n = \phi$, the empty set. The only relation from $\{0\}$ to 0 is ϕ, the empty set. But this is not a function since its domain is not $\{0\}$. So, the assertion is vacuously true.
Inductive Step :Assume that the statement is true for n and let $f : (n+2) = \{0, 1, \ldots, n, n+1\} \longrightarrow \{0, 1, \ldots, n\} = (n+1)$ be one to one. We have two cases, depending on whether n is in the range of f or not.
Case 1: n is not in the range of f. Then, the restriction of f to $n+1 = \{0, 1, \ldots, n\}$ is one to one and its range is a subset of $\{0, 1, \ldots, n-1\} = n$. This contradicts the induction hypothesis.
Case 2: n is in the range of f. Let $i \in (n+2) = \{0, 1, \ldots, n, n+1\}$ be such that $f(i) = n$. We define a function $g : (n+2) = \{0, 1, \ldots, n, n+1\} \longrightarrow \{0, 1, \ldots, n\} = (n+1)$ by

$g(i) = f(n+1)$
$g(n+1) = f(i)$
$g(j) = f(j)$ if $j \neq i$ and $j \neq n+1$

Since f is injective, so is g. The restriction of g to $n+1$ is one-to-one and its range is a subset of $n = \{0, 1, \ldots, n-1\}$. Again, this contradicts the induction hypothesis.

Grading Criteria:

1. Just trying (listing the cases) : 3 points
2. Basis: 3 points
3. The inductive step: 14 points

0.3in 3.0 The split by cases: 1 point
   3.1 Case 1: 3 points
   3.2 Case 2: 10 points
      3.2.1 The construction of g: 7 points
      3.2.2 The rest of the Case 2 proof: 3 points

Question 3 (30 points)

1. a 2. d 3. d 4. b 5. b 6. c 7. b 8. b 9. b 10. c 11. d 12. d 13. a 14. b 15. a

Proof:

We need to prove $\mathcal{P}[F]$.

$\mathcal{P}[\mathcal{F}]$ Every nonempty suffix of F with $n[(, S] = n[), S]$ is a subformula of F.

So we have to show that every S satisfies $(*)$.

(*) if S is a no-empty suffix of F with $n[(, S] = n[), S]$ is a subformula of F.

Case 1: F is an atom.

The only suffixes of F are $S = \lambda$ and $S = F$. If $S = \lambda$, the statement is vacuously true. If $S = F$, S is a formula, so again $(*)$ is true.

Case 2: $F = \neg G$.

The suffixes of F are either suffixes of G or F itself. If S is a suffix of G, then S satisfies $(*)$ by IH. If $S = F$, then $(*)$ is satisfied because S is a formula.

Cases 3,4,5,6: $F = (GCH)$ where C is a binary connective.

The suffixes of F fall into 4 subcase below:

Subcase 3.1: $S = \lambda$.

Then $(*)$ is vacuously true.

Subcase 3.2: $S = J)$ where J is a suffix of H.

$n[), S] = n[), J] + 1$ because $S = J)$

$> n[), J]$ drop the 1

$\geq n[(, J]$ by Lemma 2 from the class

$= n[(, J)]$

$= n[(, S]$ because $S = J)$

So, $n[), S] > n[(, S]$.

For these suffixes S, $(*)$ is vacuously true.

Subcase 3.2: $S = ICH)$ where I is a suffix of G.

$n[), S] = n[), I] + n[), H] + 1$ because $S = ICH)$

$> n[), I] + n[), H]$ drop the 1

$\geq n[(, I] + n[(, H]$ Lemma 2 from the class notes applied twice

$= n[(, ICH)$

$= n[(, S]$ because $S = ICH)$

So, $n[), S] > n[(, S]$.

For these suffixes S, $(*)$ is vacuously true.

Subcase 3.4: $S = F$. Since S is a formula, S satisfies $(*)$.

Grading Criteria:

1. Listing the cases: 2 points
2: Case 1: 2 points
3. Case 2: 4 points
4. Cases 3,4,5,6: 12 points
    4.1 the split into subcases and Subcase 3.1: 2 points
    4.2 Subcase 3.2: 4 points
    4.2 Subcase 3.3: 5 points
    4.3 Subcase 3.4: 1 point