COT 3420
Section U1
Spring 2012

EXAM # 2 ANSWERS

QUESTIONS

Question 1. (20 points)
Answers: 1. c 2. a, d 3. d 4. a 5. c 6. b 7. c 8. c 9. b 10. c
Grading Criteria: 2 points for each correct answer.

Question 2. (20 points)
Construct a derivation tree of $\square$ from
$S = \{\{A, B, C\}, \{A, \neg B, C, D\}, \{A, \neg B, C, \neg D\}, \{A, \neg C, \neg E\}, \{A, \neg C, E\}, \{\neg A, \neg D\}, \{\neg A, D, E\}, \{\neg A, D, \neg E\}\}$.

Answer: The derivation tree is shown in Figure 1.
Grading Criteria: 1. 20/7 for each correct resolution leading to $\square$.
2. -4 points for each incorrect derivation step.
3. -3 points for diagrams that are hard to follow.
4. Score = number of correct resolvents - penalties

Question 3. (20 points)
Solution
Cases 1, 2: t is a variable or a constant. Then, t is a symbol that is neither a left parenthesis, nor a comma, nor a function symbol of arity greater than 0. So,
$n[comma, t] = n[(, t] = n[arity, t] = 0$
The equality $n[arity, t] = n[(, t] + n[comma, t]$
becomes
$0 = 0 + 0$
which is true.
Case 3: $t = g(t_1, \ldots, t_m)$. The IH holds for all $1 \leq i \leq m$.
(IH) $n[arity, t_i] = n[(, t_i] + n[comma, t_i]$

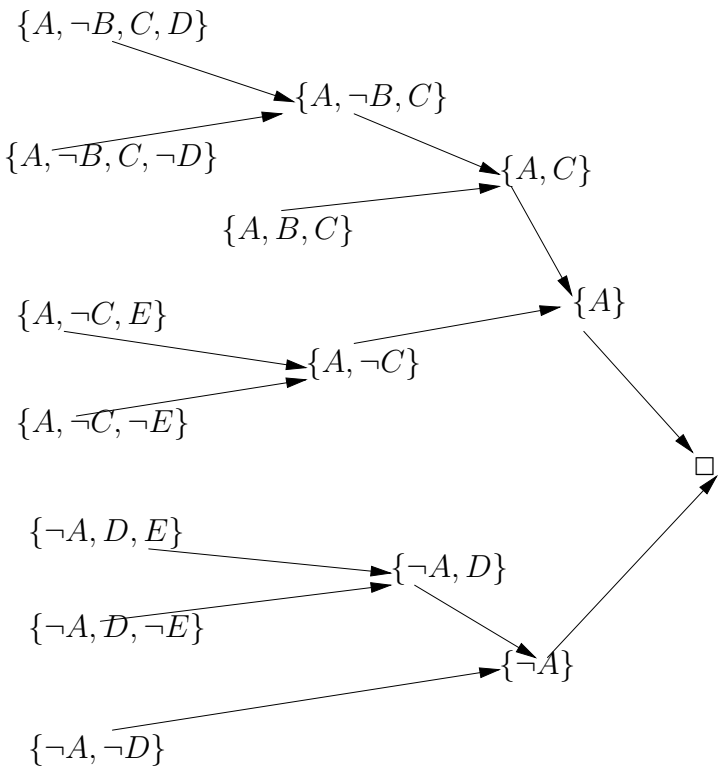

Figure 1: The Derivation Tree for Question 2

Now let us compute $n[arity, t]$.
$n[arity, t] = n[arity, g(t_1, \ldots, t_m)]$ because $t = g(t_1, \ldots, t_m)$
$= m + n[arity, t_1] + \ldots + n[arity, t_m]$
$= m + n[(, t_1] + n[comma, t_1] + \ldots + n[(, t_m] + n[comma, t_m]$ by IH
$= 1+m-1+n[(, t_1]+n[comma, t_1]+\ldots+n[(, t_m]+n[comma, t_m]$ m = 1 + m-1
$= \{1+n[(, t_1]+\ldots n[(, t_m]\}+\{m-1+n[comma, t_1]+\ldots+n[comma, t_m]\}$ by grouping
$= n[(, g(t_1, \ldots, t_m)] + n[arity, g(t_1, \ldots, t_m)]$
$= n[(, t] + n[arity, t]$ because $t = g(t_1, \ldots, t_m)$

Grading Criteria:

1. Listing the cases: 2 points
2. Cases 1 and 2: 4 points
3. Case 3: 14 points
    - 3.1. Listing the IH: 3 points
    - 3.2. The derivation: 9 points
    - 3.3: The reasons: 2 points
4. Just trying: 2 points

Question 4. (20 points)
Prove that the set of connectives $T = \{((F_1 \vee \neg F_2) \wedge \neg F_3)\}$ is adequate.

Proof: Let $\phi(F_1, F_2, F_3) = (F_1 \vee \neg F_2) \wedge \neg F_3$. Since $S = \{\neg F, F \wedge G\}$ is adequate, we need to find formulas that use only ϕ, equivalent to $\neg$ and $\wedge$.

1. $\phi(F_1, F_1, F_1)$
$= (F_1 \vee \neg F_1) \wedge \neg F_1$ definition of ϕ
$\equiv \mathbf{T} \wedge F_1$ tautology law
$\equiv F_1$ tautology law
So, $\neg F \equiv \phi(F, F, F)$.

2. $\phi(F_1, F_2, \phi(F_2, F_2, F_2)) = (F_1 \vee \neg F_2) \wedge \neg\phi(F_2, F_2, F_2))$ definition of ϕ
$\equiv (F_1 \vee \neg F_2) \wedge \neg\neg F_2$ Part 1
$\equiv (F_1 \vee \neg F_2) \wedge F_2$ $\neg\neg$-elimination
$\equiv (F_1 \wedge F_2) \vee (\neg F_1 \wedge F_1)$ distributivity
$\equiv (F_1 \wedge F_2) \vee \Box)$ contradiction law

$\equiv (F_1 \wedge F_2)$ contradiction law

We have that $\phi(F, G, \phi(G, G, G)) \equiv (F \wedge G)$, and the proof is finished.

Grading Criteria:

1. 8 points for finding a T-meta-formula equivalent to $\neg F$. Out of these, the proof is worth 2 points.

2. 12 points for finding a T-meta-formula equivalent to $(F \wedge G)$. Out of these, the proof is worth 5 points, including 2 points for the reasons.

3. 2-3 points for just trying.

Question 5. (20 points)

$F = \neg[(A \vee \neg B \vee C) \longleftrightarrow \neg(B \wedge C \wedge D)]$

$\equiv \neg[[(A \vee \neg B \vee C) \longrightarrow \neg(B \wedge C \wedge D)] \wedge [\neg(B \wedge C \wedge D) \longrightarrow (A \vee \neg B \vee C)]]$ $\longleftrightarrow$-elim; line 1

$\equiv \neg[[\neg(A \vee \neg B \vee C) \vee \neg(B \wedge C \wedge D)] \wedge [\neg\neg(B \wedge C \wedge D) \vee (A \vee \neg B \vee C)]]$ $\longrightarrow$-elim; line 2

$\equiv \neg[\neg(A \vee \neg B \vee C) \vee \neg(B \wedge C \wedge D)] \vee \neg[\neg\neg(B \wedge C \wedge D) \vee (A \vee \neg B \vee C)]$ De Morgan's law; line 3

$\equiv [\neg\neg(A \vee \neg B \vee C) \wedge \neg\neg(B \wedge C \wedge D)] \vee [\neg\neg\neg(B \wedge C \wedge D) \wedge \neg(A \vee \neg B \vee C)]$ De Morgan's law twice; line 4

$\equiv [(A \vee \neg B \vee C) \wedge B \wedge C \wedge D] \vee [\neg(B \wedge C \wedge D) \wedge \neg A \wedge \neg\neg B \wedge \neg C]$ $\neg\neg$-elim 3 times, generalized DeMorgan's; line 5

$\equiv (B \wedge C \wedge D) \vee [(\neg B \vee \neg C \vee \neg D) \wedge \neg A \wedge B \wedge \neg C]$ absorbtion, generalized DeMorgan's, $\neg\neg$-elim; line 6

$\equiv (B \wedge C \wedge D) \vee (\neg A \wedge B \wedge \neg C)$ absorbtion; line 7

$\equiv (B \vee \neg A) \wedge (B \vee B) \wedge (B \vee \neg C) \wedge (C \vee \neg A) \wedge (C \vee B) \wedge (C \vee \neg C) \wedge (D \vee \neg A) \wedge (D \vee B) \wedge (D \vee \neg C)$ generalized distributivity; line 8

$\equiv (\neg A \vee B) \wedge B \wedge (B \vee \neg C) \wedge (\neg A \vee C) \wedge (B \vee C) \wedge (\neg A \vee D) \wedge (B \vee D) \wedge (\neg C \vee D)$ ordering the clauses, idempotency, tautology removal; line 9

$\equiv B \wedge (\neg A \vee C) \wedge (\neg A \vee D) \wedge (\neg C \vee D)$ abssorbtion; line 10

Grading Criteria: All lines are worth 2 points except for line 1 (1 point) and 8 (3 points).

If you did not write the reasons you loose 2 points

If you do not separate the formulas by $\equiv$ you loose 2 points