COT 3420
SUMMER A 2003
Section 1

EXAM # 1

INSTRUCTIONS

1. The exam is open book, open notebook.
2. There are 5 questions on the test, for a total of 75 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
4. If you do not understand the meaning of a question ask me during the test.
5. You have 1 hour to work on the test.
6. Write all your answers on the exam sheet.
7. Write your name below.

NAME: _____

QUESTIONS

Question 1.(20 points)

For each of the following relations select the string that provides the most accurate description. There is no penalty for wrong guessing, but choose only one answer.

1. If 0.1.1 is in the domain of the tree t, then ... must also be in the domain.
   a. 1
   b. 0.1.2
   c. 0.1.1.0
   d. 0.1.0

2. The string ... is a formula.
   a. $(P \wedge P_2)$
   b. $P_2 \longrightarrow P_0$
   c. $(P_0 \longleftrightarrow P_4)$

d. $(\neg P_3)$

3. A formula cannot end with ...
   a. P_2
   b. (
   c.)
   d. $\neg P_3$

4. The string ... is not a suffix of **Marcos**.
   a. λ
   b. **M**
   c. **s**
   d. **Marcos**

5. Let $F = (X \wedge Y)$ be a formula. Then ...
   a. Y must be a formula.
   b. $n[(, Y] \leq n[), Y]$.
   c. $n[(, Y] \geq n[), Y]$.
   d. X must be a formula.

6. $n[c, accent] =$...
   a. 0
   b. 1
   c. 2
   d. 3

7. The second occurrence of a in *babaca* occurs at index ...
   a. 1
   b. 2
   c. 3
   d. 4

8. Let G and H be formulas, and let $F = (G \wedge H)$. If I is a subformula of F and $I \neq F$, then ...
   a. I must be a subformula of G or a subformula of H.
   b. I must be a subformula of G.
   c. I must be a subformula of H.

d. I can be of the form $X \wedge Y$, where X is a suffix of G and Y is a prefix of H.

9. $\bigvee_{i=4}^{2} F_i$ represents ...
   a. $\Box$.
   b. $\mathbf{T}$.
   c. $(F_2 \vee (F_3 \vee F_4)))$.
   d. $((F_2 \vee F_3) \vee F_4)$.

10. Let t be a formula tree of height 2 and $F = \mathit{convert}(t)$. Then the length of F is less than or equal to ...
   a. 7
   b. 11
   c. 13
   d. 15

Question 2. (20 points)

Let $n[\mathit{atom}, F]$ be the number of atoms of F and $n[\mathit{con}, F]$ be the number of binary connectives of F. Prove by structural induction that $n[\mathit{atom}, F] = n[\mathit{con}, F] + 1$. Write your proof below and on the opposite page.

Question 3. (10 points)

Find the tree representation of $F = (((\neg P_1 \wedge P_3) \longleftrightarrow \neg P_7) \longleftrightarrow (P_2 \vee \neg(P_5 \longrightarrow \neg(P_4 \vee (P_9 \wedge \neg P_0)))))$.

Draw your tree on the opposite page.

Question 4. (15 points)

Prove by mathematical induction that $|\bigvee_{i=0}^{n} P_i| = 4n+1$, where $|\bigvee_{i=0}^{n} P_i|$ is the length of the string $\bigvee_{i=0}^{n} P_i$. Write your answer below.

Question 5. (10 points)

Find all subformulas of $F = (\neg((P_1 \vee \neg P_2) \longrightarrow (P_2 \wedge \neg P_3)) \longleftrightarrow \neg(P_0 \wedge (P_4 \longleftrightarrow P_7)))$.

Write your answer below.