COT 3420
Section U1
Summer 2005

EXAM # 1

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 5 questions on the test, for a total of 116 points.
3. For the multiple choice question, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour to complete the exam.
5. Mark the answers to questions 1 and 4 on the exam paper. Write the answers to the other questions on the blank sheets.
6. No talking to each other during the test!
7. Write your name below.

NAME: _____

QUESTIONS

Question 1. (36 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. Let F and G be two formulas. If $F \models G$, then ...
   a. $Mod[F] \subseteq Mod[G]$
   b. $Con[F] \subseteq Con[G]$
   c. $\neg F \models \neg G$
2. If 1.3 is a Dewey address in the tree t, then ... must be an address in the tree.
   a. 2
   b. 1.3.1
   c. 1.2.1

d. 1.2

3. Let $F \models G$. Then ...
   a. $\models (F \longrightarrow G)$
   b. $\models (G \longrightarrow F)$
   c. $\models (F \vee G)$
4. If $F \longrightarrow G$ is satisfiable, then ... is also satisfiable.
   a. $F \vee G$
   b. $\neg F \vee G$
   c. $F \vee \neg G$
   d. $\neg F \vee \neg G$
5. The string ... is not a prefix of **Granpa**.
   a. λ
   b. **Gran**
   c. **pa**
   d. **Granpa**
6. ... is always true.
   a. $F \vee G \models F$
   b. $F \models (F \wedge G)$
   c. $F, \neg F \models G$
   d. $F, G \models (F \longrightarrow \neg G)$
7. $\bigwedge_{i=3}^{1} F_i = \ldots$
   a. $\Box$.
   b. **T**.
   c. $((F_1 \wedge F_2) \wedge F_3)$.
   d. $(F_1 \wedge (F_2 \wedge F_3))$.
8. If $F \longrightarrow G$ is a tautology, then ...
   a. G is satisfiable.
   b. F is unsatisfiable.
   c. $F = G$.
   d. $\neg G \longrightarrow \neg F$ is a tautology.
9. Let F be a satisfiable formula. Then F has ...
   a. finitely many models.
   b. countably many models.
   c. uncountably many models.
10. If $F \wedge G$ is unsatisfiable, then ...
    a. F is unsatisfiable.
    b. G is unsatisfiable.
    c. $F \models \neg G$

11. Let F be a formula of length 20. Then ...
    a. it has at most 20 subformulas.
    b. it has at most 15 subformulas.
    c. it has at most 10 subformulas.
    d. it has no subformulas, because no formula has length 20.
12. There ... tautologies.
    a. finitely many
    b. countably many
    c. uncountably many

Question 2. (25 points)

Prove that for all formulas F, $|F| = n[\neg, F] + 4 * n[atom, F] - 3$. Recall that $n[atom, F]$ is the number of atom occurrences in F and $|F|$ is the length of F.

Question 3. (15 points)

Draw the formula tree of $F = ((((P_0 \vee \neg P_1) \wedge \neg P_3) \longrightarrow ((P_3 \longrightarrow P_1) \longleftrightarrow \neg(P_5 \vee P_2))) \longleftrightarrow \neg((P_3 \wedge P_4) \vee (P_7 \longrightarrow \neg(P1 \wedge P_2))))$.

Question 4. (15 points)

Write the formula that corresponds to the tree from Figure 1.

Write your answer below.

Question 5. (25 points)

Prove or disprove: If F and G have no atoms in common, $F \longrightarrow H$ and $G \longrightarrow H$ are satisfiable, then $(F \vee G) \longrightarrow H$ is satisfiable.

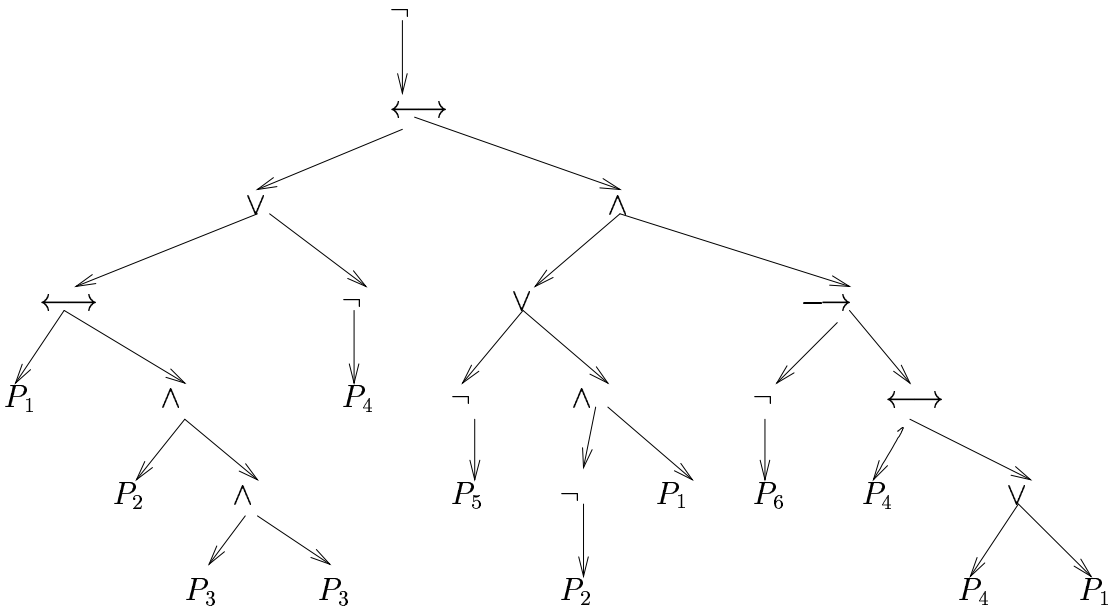

Figure 1: The tree for Question 4