COT 3420
Section U1
Summer 2005

EXAM # 2

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 5 questions on the test for a total of 126 points.
3. There is a bonus question, worth 15 points. You may use it to make up the points lost on the other 5 questions.
4. For the multiple choice question, there is no penalty for wrong guessing. For the others, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour and 15 minutes to complete the exam.
5. Mark the answers to question 1 on the exam paper. Write the answers to the other questions on the blank sheets.
6. No talking to each other during the test!
7. Write your name below.

NAME: _

QUESTIONS

Question 1. (36 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. If $C \in Res^n[S]$, then ...
   a. C has a derivation tree of height n.
   b. C cannot have derivation trees of height greater than n.
   c. C has a derivation tree of height less than or equal to n.
2. If $Res^n[S] = Res^{n+1}[S]$, then ...
   a. $Res^*[S]$ has no derivation trees of height $n + 1$.
   b. $Res^*[S]$ has no minimal derivation trees of height $n + 1$.

c. $Res^*[S]$ has no derivation trees of height greater than n.

3. A clause is **negative** when it is not empty and contains only negations of atoms. If $\Box \notin S$, and S has no negative clauses, then S ...

a. is satisfiable.

b. is unsatisfiable

c. can be satisfiable or unsatisfiable.

4. A formula has ... CNF's.

a. finitely many

b. countably many

c. uncountably many

5. If H is a CNF for both F and G, then ...

a. $F = G$.

b. $F \equiv G$.

c. $F \not\equiv G$.

d. $F \neq G$.

6. Let S be the set of all non-equivalent clauses that can be formed with the atoms $P_1, \ldots, P_n$. The size of S is approximated by ...

a. $2n$.

b. 2^n.

c. 3^n.

d. 4^n.

7. Let C and D be two clauses such that $C \subseteq D$. Then ...

a. $C \models D$.

b. $D \models C$.

c. $C \models D$ or $D \models C$.

d. for some $C \subseteq D$ neither $C \models D$ nor $D \models C$ holds.

8. Let S be an infinite set of non-equivalent formulas. If S is unsatisfiable, then it has ... subsets that are both finite and unsatisfiable.

a. finitely many

b. countably many

c. uncountably many

9. The number of non-equivalent formulas that can be formed with n atoms is ...

a. 2^n.

b. 3^n.

c. 4^n.

d. 2^{2^n}.

10. The clause set S is minimally unsatisfiable if every proper subset of S is satisfiable. Let S be a minimally unsatisfiable set, t a derivation tree of $\square$ from S, and T the set of all clauses that are leaves in t. Then ...

a. $S = T$.

b. $S \subseteq T$.

c. $T \subseteq S$.

11. Let $S = \{C_0, C_1, \ldots, C_n, \ldots\}$ be an infinite set of clauses. If S is satisfiable, then ...

a. for every n there is a number $m > n$ such that $\bigwedge_{i=0}^{m} F_i$ is satisfiable.

b. there is some n such that $Res^*[S] = Res^n[S]$.

c. for every literal L that occur in S, its complement does not occur in S.

12. Let S be an unsatisfiable set of clauses and T a minimally unsatisfiable subset of S. If $\square \notin T$, then T contains ...

a. tautologies.

b. clauses that are subsumed by other clauses of T.

c. clauses that cannot be unified with other clauses of T.

d. literals L such that $\bar{L}$ is not in T.

e. unifiable clauses.

Question 2. (20 points)

Display a derivation tree of $\square$ from $S = \{\{A, B, C\}, \{A, \neg B\}, \{\neg C, D\}, \{\neg C, \neg D\}, \{\neg F, \neg G\}, \{\neg F, G\}, \{\neg A, F\}\}$.

Question 3. (25 points)

Prove, by structural induction, that the connective $F \longrightarrow \neg G$ is adequate.

Question 4. (25 points)

Let S be a set of clauses with n atoms. Prove that S is unsatisfiable iff $\square \in Res^n[S]$.

Hint: The set $S_{A=0}$ is obtained from S by deleting all clauses that contain $\neg A$ and then deleting A from the remaining clauses. The set $S_{A=1}$ is obtained from S by deleting all clauses that contain A and then deleting $\neg A$ from the remaining clauses. So, for $S = \{\{A, B\}, \{A, \neg B\}, \{A, \neg A\}, \{\neg A, C\}, \{\neg A, \neg C\}\}$, $S_{A=0} = \{\{B\}, \{\neg B\}\}$ and $S_{A=1} = \{\{C\}, \{\neg C\}\}$.

Then use the proposition that says that S is unsatisfiable iff both $S_{A=0}$ and $S_{A=1}$ are unsatisfiable.

Question 5 (20 points)

Find a CNF for $F = \neg[((A \vee B) \wedge C) \longleftrightarrow \neg(B \vee D)]$.

Show your work.

Bonus (15 points)

Let F be a formula with atoms $P_1, \ldots, P_n$ and let S be the set of all clauses C with literals in the set $\{P_1, \neg P_1, P_2, \neg P_2, \ldots, P_n, \neg P_n\}$ that satisfy the relation $F \models C$. Prove that $S \equiv F$, i.e. every model of S is a model of F and viceversa.