COT 3420
Section U1
Summer 2005

FINAL EXAM

INSTRUCTIONS

1. The test is closed book, closed notebook.
2. There are 6 questions on the test, for a total of 95 points.
3. You have 1 hour and 15 minutes to complete the exam.
4. Write all answers on the blank sheets.
5. No talking to each other during the test!
6. Write your name below.

NAME: _

QUESTIONS

Question 1. (10 points)

Rectify the formula

$F = \forall x(P(f(x), y) \vee \forall y(\exists x P(x, y) \vee \neg \forall z P(x, g(y))))) \wedge \forall y(\exists x \neg P(y, x) \vee \neg P(y, x))$.

Question 2 (10 points)

Skolemize the formula

$F = \exists x \forall y(P(x, y) \vee \exists z P(z, f(y))) \wedge \forall u[\forall v(\exists w(\neg P(u, w) \vee \neg P(w, v)) \wedge \exists w_1 \neg P(w_1, u)) \vee \exists w_2 \neg P(w_2, u)]$

Question 3. (20 points)

Prove by structural induction that every formula F has a prenex form. You may use the relabeling lemma and the semantic equivalences from the book, but do not use the Skolemization algorithm.

Question 4. (20 points)

Derive $\Box$ from $S = \{\{P(x, f(x), y), P(y, z, y), Q(x, y, z)\}, \{R(f(x), g(y)), R(y, g(z)), \neg Q(x, x, z)\}, \{\neg P(g(x), y, z), Q(z, z, f(u))\}, \{\neg R(y, g(x)), \neg Q(u, u, y)\}\}$.

Do the minimal number of resolutions.

Question 5 (15 points)

List $D(F,2)$ for the Skolem form $F = \forall x \forall y((\neg P(f(a),y) \vee \neg P(x,g(y))) \wedge (P(a,h(x)) \vee P(y,g(a))) \wedge P(a,f(y)))$.

Question 6 (20 points)

The *subsumption* is a simplification rule widely used in theorem proving. We say that a clause $C_1 = \{L_1, ..., L_n\}$ subsumes a clause $C_2 = \{M_1, ..., M_m\}$ if there is a substitution $s = [z_1/t_1, ..., z_r/t_r]$, where z_1, z_2,...z_r are variables in C_1, such that $s[C_1] \subseteq C_2$.

For example, $\{P(x,y), P(y,x)\}$ subsumes $\{P(x,x), Q(x)\}$ because $\{P(x,y), P(y,x)\}[y/x] = \{P(x,x), P(x,x)\} \subseteq \{P(x,x), Q(x)\}$.

Let z_1,...,z_r be the variables of C_1, and y_1, ...,y_q be the variables of C_2. Prove that if C_1 subsumes C_2 then

$\forall z_1 .... \forall z_r C_1 \wedge \forall y_1 ... \forall y_q C_2 \equiv \forall z_1 .... \forall z_r C_1$.