COT 3420
Section U2
Fall 2005

EXAM # 1

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 5 questions on the test, for a total of 115 points.
3. For the multiple choice question, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour to complete the exam.
5. Mark the answers to question 1 on the exam paper. Write the answers to the other questions on the blank sheets.
6. No talking to each other during the test!
7. Write your name below.

NAME: _____

QUESTIONS

Question 1. (40 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. Let F and G be two formulas. If $Con[F] = Con[G]$, then ...
   a. $F = G$
   b. $F \equiv G$
   c. sometimes $F \equiv G$ and sometimes $F \not\equiv G$
2. If 2.3 is a Dewey address in the tree t, then ... must be an address in the tree.
   a. 2.3.0
   b. 2.2.0
   c. 2.2
   d. 1.0

3. Assume that $F \models G$ and $F \models \neg G$. If H is a formula, then ...
   a. $F \models H$.
   b. $G \models F$.
   c. $H \models F$.
   d. $G \models H$.
4. If $F \longleftrightarrow \neg G$ is satisfiable then ... is also satisfiable.
   a. $\neg F \vee G$
   b. $F \vee G$
   c. $F \vee \neg G$
5. The string ... is not a suffix of **Markey**.
   a. λ
   b. **Markey**
   c. **key**
   d. **Mar**
6. The relation $F \models \neg F$ ...
   a. is not possible.
   b. is true only when F is a tautology.
   c. is true only when F is an atom.
   d. is true only when F is unsatisfiable.
7. $\bigvee_{i=1}^{3} F_i = \ldots$
   a. $\Box$.
   b. **T**.
   c. $((F_1 \vee F_2) \vee F_3)$.
   d. $(F_1 \vee (F_2 \vee F_3))$.
8. If $F \longrightarrow G$ is satisfiable, then ...
   a. G is satisfiable.
   b. F is unsatisfiable.
   c. $(F \wedge H) \longrightarrow (G \wedge H)$ is satisfiable.
   d. $\neg F \longrightarrow \neg G$ is satisfiable.
9. Let S be a set of formulas that has a countably infinite set of models. Then, ...
   a. all atoms are in S, except for a finite number.
   b. S contains an infinite number of atoms.
   c. S must be infinite.
   d. S can be finite or infinite.
10. Let S be an unsatisfiable set of formulas, and T a set of formulas. Then, ... is unsatifiable.
   a. $S \cap T$

b. $S \cup T$
c. $S - T$
d. $T - S$

11. If $Res^*[S] = Res^n[S]$, then ...
a. there are no derivation trees of height $n + 1$.
b. all derivation trees must have height less than or equal to n.
c. there are no minimal derivation trees of height $n + 1$.

12. If S is a set of clauses, then ...
a. $Res^*[Res^*[S]] = Res^*[S]$.
b. $Res^*[Res^*[S]] \neq Res^*[S]$.
c. $Res^*[Res^*[S]]$ does not exist.

13. Let H be a CNF for F. If H is not a CNF for G, then ...
a. $F \equiv G$.
b. $F \not\equiv G$.
c. sometimes $F \equiv G$ and sometimes $F \not\equiv G$.

14. There are ... sets of formulas S that satisfy the relation $S = Con[S]$.
a. finitely many
b. countably many
c. uncountably many

15. The set of connectives ... is adequate.
a. $\{\longrightarrow\}$.
b. $\{\vee, \wedge\}$.
c. $\{\vee, \longrightarrow\}$.
d. $\{\longrightarrow, \neg\}$.

16. The domain of $\boxed{\wedge}$ is ...
a. the set $\{0, 1\} \times \{0, 1\}$.
b. the set $\{0, 1\}$.
c. the set of all formulas, FORM.
d. FORM $\times$FORM.

17. Let S be an infinite set of non-equivalent formulas. If S is unsatisfiable, then it has ... unsatisfiable subsets.
a. finitely many
b. countably many
c. uncountably many

18. Let us assume that clauses C_1 and C_2 have two distinct resolvents, R_1 and R_2. Then ...
a. $C_1 = C_2$.
b. at least one of C_1, C_2 must be a tautology.

c. both R_1 and R_2 must be tautologies.

d. this is not possible.

19. The clause set S is minimally unsatisfiable if every proper subset of S is satisfiable. Let $S \neq \{\Box\}$ be a minimally unsatisfiable set. Then,

a. every clause must be unifiable with some other clause.

b. there is an atom A such that both $\{A\}$ and $\{\neg A\}$ are clauses in S.

c. S can contain tautologies.

d. S cannot contain unit clauses.

20. Let $S = \{C_0, C_1, \ldots, C_n, \ldots\}$ be an infinite set of clauses. If S is unsatisfiable, then ...

a. S contains a finite number of atoms.

b. there is some n such that $Res^*[S] = Res^n[S]$.

c. for every literal L that occur in S, its complement must also occur in S.

d. there is some n such that $\{C_0, C_1, \ldots, C_n\}$ is unsatisfiable.

Question 2. (15 points)

Display a derivation tree of $\Box$ from $S = \{\{\neg B, C, D\}, \{\neg A, F\}, \{\neg B, \neg D, \neg E\}, \{\neg B, \neg C, D\}, \{A, B\}, \{\neg B, \neg D, E\}, \{\neg A, \neg F\}\}$.

Question 3. (20 points)

Prove, by structural induction, that every non-empty suffix of F that has an equal number of left and right parentheses is a formula.

Question 4 (20 points)

Apply the CNF algorithm from the book to find a CNF for $F = \neg\{[(A \vee B) \longleftrightarrow C] \wedge [(A \wedge C) \longleftrightarrow D]\}$.

Show your work.

Question 5. (20 points)

Prove or disprove: Let F, G, H, I be 4 formulas such that $(F \wedge G) \wedge H$, $(G \wedge H) \wedge I$, $(H \wedge I) \wedge F$, and $(I \wedge F) \wedge G$ are satisfiable. Then $(F \wedge G) \wedge (H \wedge I)$ is satisfiable.

First write proof or disproof, and then continue. If you don't make a choice or if you indicate both choices, you get 0 points.

PS I will add 8 points to the score of the students who took the test. The promisses must be kept ...