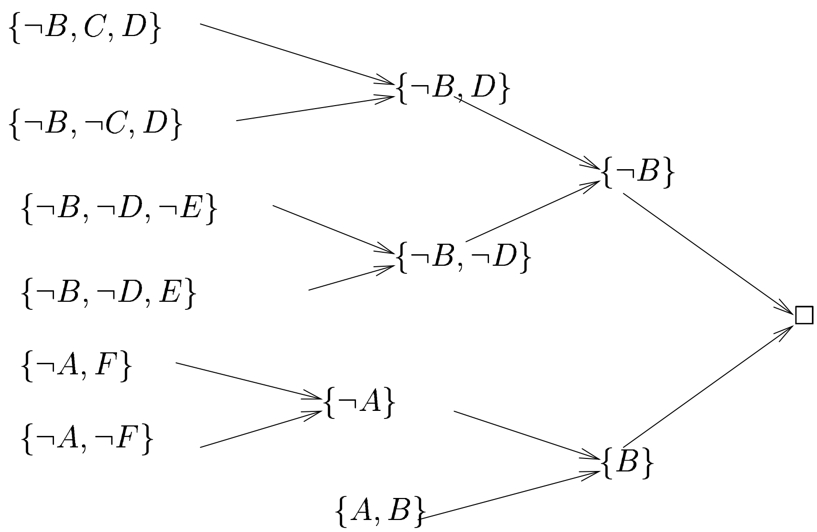

Figure 1: The tree for Question 2

COT 3420
Section U2
Fall 2005

EXAM # 1 SOLUTIONS

QUESTIONS
Question 1. (40 points)

1. b 2. c 3. a 4. b 5. d 6. d 7. c 8. c 9. a 10. b 11. c 12. a 13. b 14. c 15. d 16. a 17. c 18.c 19. a 20.d

Grading Criteria

2 points for each correct answer.

Question 2. (15 points)

The tree is shown in Figure 1.

Grading Criteria

1. 2.5 points for each correct resolvent, up to 6.
2. -2 points if the $\Box$ is not reached in 6 steps.
3. There is no credit for inferences whose parents are not drived correctly, i.e. there was an error on the path from a leaf to a parent.
4. -1.5 points for each misdirected arrow.

Question 3. (20 points)

Prove, by structural induction, that every non-empty suffix of F that has an equal number of left and right parentheses is a formula.

Proof: We need to prove the proposition below.

If S is a suffix of F, $S \neq \lambda$ and $n[(, S] = n[), S]$, then S is a formula.

Case 1: F is an atom, say P_i. The only non-empty suffix of F is F itself, a formula.

Case 2: $F = \neg G$. Let S be a suffix of F that satisfies the conditions $S \neq \lambda$ and $n[(, S] = n[), S]$. We have 2 subcases.

Subcase 2.1: S is a suffix of G. Then S is a formula by the IH applied to G.

Subcase 2.2: $S = F$. Then S is a formula.

Cases 3,4,5,6: $F = (GCH)$, where $C$is one of the binary connectives $\vee$, $\wedge$, $\longrightarrow$, $\longleftrightarrow$. Let S be a suffix of F that satisfies the conditions $S \neq \lambda$ and $n[(, S] = n[), S]$. We have 3 subcases.

Subcase 3.1: $S = J)$ where J is a suffix of H. We'll show that the condition $n[(, S] = n[), S]$ is false, so the proposition is vacuously true.

$n[), S] = n[), J)]$ $S = J)$
$= n[), J] + 1$
$> n[), J]$
$\geq n[(, J]$ By Lemma 1.2.8 applied to the suffix J of H
$= n[(, J)]$
$= n[(, S]$ $S = J)$

So, $n[), S] > n[(, S]$.

Subcase 3.2: $S = ICH)$ where I is a suffix of H. We'll show that the condition $n[(, S] = n[), S]$ is false, so the proposition is vacuously true.

$n[), S] = n[), ICH)]$ $S = ICH)$
$= n[), I] + n[), H] + 1$
$> n[), I] + n[), H]$
$\geq n[(, I] + n[), H]$ By Lemma 1.2.8 applied to the suffix I of G
$\geq n[(, I] + n[(, H]$ By Lemma 1.2.8 applied to the suffix H of H
$= n[(, ICH)]$
$= n[(, S]$ $S = ICH)$

So, $n[), S] > n[(, S]$.

Subcase 3.3: $S = F$. Then S is a formula.

Grading Criteria:

1. Listing the cases: 1 point

2. Case 1: 1 point

3. Case 2: 6 points (2 points for listing the subcases, 3 points for subcase 3.1, 1 point for subcase 2.2)

4. Case 3: 12 points (3 points for listing the subcases, 4 points for subcases 3.1 and 3.2, and 1 point for subcase 3.3).

Question 4 (20 points)

Find a CNF for $F = \neg\{[(A \vee B) \longleftrightarrow C] \wedge [(A \wedge C) \longleftrightarrow D]\}$.

Show your work.

$F = \neg\{[(A \vee B) \longleftrightarrow C] \wedge [(A \wedge C) \longleftrightarrow D]\}$

$\equiv \neg\{[(A \vee B) \longrightarrow C] \wedge [C \longrightarrow (A \vee B)] \wedge [(A \wedge C) \longrightarrow D] \wedge [D \longrightarrow (A \wedge C)]\}$ $\longleftrightarrow$-elim, line 1

$\equiv \neg\{[\neg(A \vee B) \vee C] \wedge [\neg C \vee (A \vee B)] \wedge [\neg(A \wedge C) \vee D] \wedge [\neg D \vee (A \wedge C)]\}$ $\longrightarrow$-elim, line 2

$\equiv \neg[\neg(A \vee B) \vee C] \vee \neg[\neg C \vee (A \vee B)] \vee \neg[\neg(A \wedge C) \vee D] \vee \neg[\neg D \vee (A \wedge C)]$ generalized DeMorgan's, line 3

$\equiv [\neg\neg(A \vee B) \wedge \neg C] \vee [\neg\neg C \wedge \neg(A \vee B)] \vee [\neg\neg(A \wedge C) \wedge \neg D] \vee [\neg\neg D \wedge \neg(A \wedge C)]$ DeMorgan's 4 times, line 4

$\equiv [(A \vee B) \wedge \neg C] \vee [C \wedge (\neg A \wedge \neg B)] \vee [A \wedge C \wedge \neg D] \vee [D \wedge (\neg A \vee \neg C)]$ $\neg\neg$-elim twice, DeMorgan's, $\neg\neg$-elim twice, DeMorgan's, line 5

$\equiv \{[(A \vee B) \wedge \neg C] \vee [\neg A \wedge \neg B \wedge C]\} \vee \{[A \wedge C \wedge \neg D] \vee [(\neg A \vee \neg C) \wedge D]\}$ grouping and ordering, line 6

$\equiv \{(A \vee B \vee \neg A) \wedge (A \vee B \vee \neg B) \wedge (A \vee B \vee C) \wedge (\neg C \vee \neg A) \wedge (\neg C \vee \neg B) \wedge (\neg C \vee C)\} \vee \{(A \vee \neg A \vee \neg C) \wedge (A \vee D) \wedge (C \vee \neg A \vee \neg C) \wedge (C \vee D) \wedge (\neg D \vee \neg A \vee \neg C) \wedge (\neg D \vee D)\}$ Generalized distributivity, line 7

$\equiv \{(A \vee B \vee C) \wedge (\neg C \vee \neg A) \wedge (\neg C \vee \neg B)\} \vee \{(A \vee D) \wedge (C \vee D) \wedge (\neg D \vee \neg A \vee \neg C)\}$ tautology law, line 8

$\equiv (A \vee B \vee C \vee A \vee D) \wedge (A \vee B \vee C \vee C \vee D) \wedge (A \vee B \vee C \vee \neg D \vee \neg A \vee \neg C) \wedge (\neg C \vee \neg A \vee A \vee D) \wedge (\neg C \vee \neg A \vee C \vee D) \wedge (\neg C \vee \neg A \vee \neg D \vee \neg A \vee \neg C) \wedge (\neg C \vee \neg B \vee A \vee D) \wedge (\neg C \vee \neg B \vee C \vee D) \wedge (\neg C \vee \neg B \vee \neg D \vee \neg A \vee \neg C)$ Generalized distributivity, line 9

$\equiv (A \vee B \vee C \vee D) \wedge (\neg C \vee \neg A \vee \neg D) \wedge (\neg C \vee \neg B \vee A \vee D) \wedge (\neg C \vee \neg B \vee \neg D \vee \neg A)$ tautology law, idempotency, line 10

$\equiv (A \vee B \vee C \vee D) \wedge (\neg A \vee \neg C \vee \neg D) \wedge (A \vee \neg B \vee \neg C \vee D)$ absorbtion, ordering, line 11

Grading Criteria

You receive credit up to the first line that contains an error.

No line is correct 1 point.

1 correct line: 2 points

2 correct lines: 3 points
3 correct lines: 5 points
4 correct lines: 7 points
5 correct line: 9 points
6 correct lines: 11 points
7 correct lines: 13 points
8 correct lines: 15 points
9 correct lines: 17 points
10 correct lines: 19 points
11 correct lines: 20 points

Question 5. (20 points)

Prove or disprove: Let F, G, H, I be 4 formulas such that $(F \wedge G) \wedge H$, $(G \wedge H) \wedge I$, $(H \wedge I) \wedge F$, and $(I \wedge F) \wedge G$ are satisfiable. Then $(F \wedge G) \wedge (H \wedge I)$ is satisfiable.

First write proof or disproof, and then continue. If you don't make a choice or if you indicate both choices, you get 0 points.

Disproof: Let $F = P_1 \wedge P_2, G = P_1 \wedge \neg P_2, H = \neg P_1 \wedge P_2, I = \neg P_1 \wedge \neg P_2$. We can go directly to the table, but we can do it by simplification.

$(F \wedge G) \wedge H = ((P_1 \wedge P_2) \wedge (P_1 \wedge \neg P_2)) \wedge (\neg P_1 \wedge P_2)$
$\equiv (P_1 \wedge (P_2 \vee \neg P_2)) \wedge (\neg P_1 \wedge P_2)$ by distributivity
$\equiv P_1 \wedge (\neg P_1 \wedge P_2)$ by the tautology law
$\equiv P_1 \wedge P_2$ because $U \wedge (\neg U \vee W) \equiv U \wedge W$.

In a similar fashion
$(G \wedge H) \wedge I \equiv \neg P_1 \wedge \neg P_2$, $(H \wedge I) \wedge F \equiv \neg P_1 \wedge P_2$ and $(I \wedge F) \wedge G \equiv P_1 \wedge \neg P_2$.

Each of these 4 formulas has a model. Now let us compute $(F \wedge G) \wedge (H \wedge I)$.

$(F \wedge G) \wedge (H \wedge I) \equiv ((F \wedge G) \wedge H) \wedge I$ by the associativity of $\wedge$
$\equiv P_1 \wedge P_2 \wedge (\neg P_1 \vee \neg P_2)$ evaluating $(F \wedge G) \wedge H$ and I
$\equiv P_1 \wedge P_2 \wedge \neg P_2$ we apply $U \wedge (\neg U \vee W) \equiv U \wedge W$ to P_2 and $(\neg P_1 \vee \neg P_2)$
$\equiv \Box$ by the contradiction law

Grading Criteria

1. If you wrote **Proof** you get 3 points.

2. If you wrote **Disproof** you get 7 points for the guess, 2 points for each correct value of a meta-variable, and 5 points for the proof that the picks are a counter-example. A truth table proof is acceptable, but you cannot get credit for proof unless all 4 picks are good.

PS I will add 8 points to the score of the students who took the test. The promisses must be kept ...