COT 3420
Section U2
Fall 2005

EXAM # 2

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 6 questions on the test, for a total of 85 points.
3. You have 1 hour and 15 minutes to complete the exam.
4. No talking to each other during the test!
5. If you do not understand the meaning of a question, ask me during the test!
6. Write your name below.

NAME: _

QUESTIONS

Question 1. (10 points)

Rectify the formula

$F = [\forall x \forall y \exists z P(x,z) \wedge \forall x \forall y \forall z (\neg P(x,y) \vee P(y,x))] \wedge P(x,x)$.

Write your answer below.

Question 2 (10 points)

Skolemize the formula

$F = \exists x_1 \forall x_2 \exists x_3 \exists x_4 \forall x_5 \exists x_6 \forall x_7 \exists x_8 F^M[a, x_1, f(x_2), x_2, x_3, x_4, x_5, x_6, x_7, x_8]$

where F^M is the matrix of F.

Write your answer below.

x	$f^{\mathcal{A}}[x]$
3	5
4	3
5	3

$f^{\mathcal{A}}$

$x \backslash y$	3	4	5
3	4	5	3
4	3	4	5
5	5	3	4

$g^{\mathcal{A}}$

$x \backslash y$	3	4	5
3	0	0	0
4	1	0	1
5	0	1	0

$P^{\mathcal{A}}$

x	$Q^{\mathcal{A}}[x]$
3	0
4	0
5	1

$Q^{\mathcal{A}}$

Figure 1: Tables for Question 5

Question 3. (20 points)

Prove by structural induction that the set $S = \{\neg F \wedge G, \forall x F\}$ is adequate, i.e. every FOL formula is equivalent to an S-formula. It's Ok to reduce some cases to cases that are already solved.
Write your answer on a blank sheet.

Question 4. (5 points)

Close the formula $F = \exists u \forall y F^M[x, y, z, u, v, w]$ where F^M, the matrix of F, has free occurrences of x, y, z, u, v, w.

Write your answer below.

Question 5. (20 points)

The universe of $\mathcal{A}$ is $\{3, 4, 5\}$ and the $\mathcal{A}$ interpretations of a,x, and y are $a^{\mathcal{A}} = 3$, $x^{\mathcal{A}} = 4$ and $y^{\mathcal{A}} = 5$. The tables for the functions $f^{\mathcal{A}}$ and $g^{\mathcal{A}}$ and the predicates $P^{\mathcal{A}}$ and $Q^{\mathcal{A}}$ are displayed in Figure 1.

Evaluate the terms and the formulas below. Do not show your work, just write the answer to the right of the equal sign.

1. $\mathcal{A}[f(g(x, y))] =$
2. $\mathcal{A}[g(g(a, x), y)] =$
3. $\mathcal{A}[\neg P(y, a)] =$
4. $\mathcal{A}[\forall x \neg P(a, f(x))] =$
5. $\mathcal{A}[\exists x \forall y P(x, f(y))] =$

6. $\mathcal{A}[\forall x(P(y, f(x)) \vee Q(f(x)))] =$

Question 6. (20 points)

Let x, u, v be 3 different variables and $\forall xF$ be a formula that does not have any occurrences of u and v.

Prove the consequence below.

$\forall xF \models \forall u \forall v F[x/f(u,v)]$.

Write your answer on a blank sheet of paper.