COT 3420
Section U2
Fall 2005

FINAL EXAM

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 6 questions on the test, for a total of 110 points.
3. For the multiple choice question, there is no penalty for wrong guessing. For the others, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 2 hours and 15 minutes to complete the exam.
5. Mark the answers to question 1 on the exam paper. Write the answers to the other questions on the blank sheets.
6. No talking to each other during the test!
7. Write your name and panther id below.

NAME: _
Panther Id: _

QUESTIONS

Question 1. (15 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. $[x/u, u/x, v/y, w/v] \diamond [x/y, y/x, z/u, v/w] = \ldots$
   a. $[x/y, y/u, z/x, u/x, w/v]$
   b. $[x/y, y/u, z/x, v/y, w/v]$
   c. $[x/y, y/u, z/x, u/x]$
   d. $[x/u, y/x, z/u, u/y, v/x]$

2. $\sigma = \ldots$ is a solution of $[x/f(g(u)), y/g(u), z/a] = [y/g(u), z/b, v/w, w/v] \diamond \sigma$.
   a. $[x/f(y), z/a]$
   b. $[x/f(y), z/b, v/w]$

c. $[x/f(y), z/a, v/w, w/v]$
d. $[x/f(y), z/b, v/w, w/v]$

3. ...is unifiable.
a. $\{P(x, f(x)), P(u, v), P(v, u)\}$
b. $\{P(f(x), y), P(z, u), P(a, v)\}$
c. $\{P(f(x), y), P(z, g(u)), P(v, h(w))\}$.
d. $\{P(x, y), P(f(y), z), P(v, g(w))\}$.

4. Let F be a formula and G be the output of the Skolem algorithm applied to F. Let $\mathcal{A}$ be a model of F. Then, G ...
a. has a model with the same universe as $\mathcal{A}$.
b. does not have a model with the same universe as $\mathcal{A}$.
c. G may or may not have a model with the same universe as $\mathcal{A}$.

5. ...is a unifier of $\{P(f(x), y), P(z, g(u)), P(u, v)\}$
a. $\sigma = [y/g(u), z/f(x), u/f(x), v/g(u)]$
b. $\sigma = [y/g(f(x)), z/f(x), u/f(x), v/g(u)]$
c. $\sigma = [y/f(g(a)), z/f(a), u/f(a)), v/g(f(a))]$
d. $\sigma = [y/f(g(x)), z/f(x), u/f(x), v/g(f(x))]$

6. The substitution ...is an mgu of $S = \{P(x, y), P(a, z), P(u, g(v))\}$.
a. $[x/a, u/a, y/g(v), z/g(v)]$
b. $[x/a, u/a, y/g(w), z/g(w), v/w]$
c. $[x/a, u/a, y/g(a), z/g(a)]$
d. $[x/a, u/x, y/z, z/g(v)]$

7. Let S be a set of FOL clauses. If all its clauses contain at least one atomic formula, then ...
a. S is satisfiable.
b. S is unsatisfiable.
c. S can be satisfiable or unsatisfiable.

8. Let S be a finite set of FOL clauses. If S is unsatisfiable then there is some $n \in N$ such that ...
a. $Res^n[S] = Res^{n+1}[S]$.
b. $Res^n[S] = Res^*[S]$.
c. $\Box \in Res^n[S]$.

9. Let S be an infinite set of FOL clauses. Then the Herbrand universe of S ...
a. is finite.
b. is countable.
c. can be uncountable.

10. The relation ...is not always true.

a. $\forall x(F \wedge G) \equiv \forall xF \wedge \forall xG$
b. $\exists xF \equiv_s F$
c. $F \equiv_s \neg F$
d. $\neg \forall xF \equiv \exists x \neg F$

Question 2. (15 points)
Construct a derivation tree of $\Box$ from $S = \{\{P(x,y), P(x,z), Q(x,y,z)\}, \{\neg P(x, f(x)), Q(x,y,y)\}, \{R(x,y), R(a,x), \neg S(x,y)\}, \{\neg R(x,y), \neg S(y,x)\}, \{S(x,y), S(y,z), \neg Q(x, f(y), f(z))\}\}$.

Question 3. (15 points)
Let $P = \forall x \forall y (P(x, f(y,x)) \wedge (\neg P(f(x,x), y) \vee \neg P(f(x,y), f(y,x))))$.
Write $E(F, 2)$. You may use F^M for $P(x, f(y,x)) \wedge (\neg P(f(x,x), y) \vee \neg P(f(x,y), f(y,x)))$.

Question 4 (20 points)
Let C_1 and C_2 be two clauses that have no variables in common, $\{A_1, \ldots, A_n\}$ be a set of atoms of C_1 and $\{\neg B_1, \ldots, \neg B_m\}$ be a set of literals of C_2. Assume that $S = \{A_1, \ldots, A_n, B_1, \ldots B_m\}$ is unifiable.
a. Show the sets $S_1 = \{A_1, \ldots, A_n\}$ and $S_2 = \{B_1, \ldots, B_m\}$ are unifiable.
b. Let s_1 be a mgu of S_1 and s_2 a mgu of S_2. Show that $\{s_1[A_1], s_2[B_1]\}$ is unifiable.
c. Let s a mgu of $\{s_1[A_1], s_2[B_1]\}$ and σ a mgu of S. Show that there is a substitution π such that $\sigma = pi \diamond s$.

Question 5. (20 points)
Prove The Compactness Theorem for FOL: A set of formulas S is unsatisfiable iff it has a finite unsatisfiable subset. Feel free to use any theorem from the book, except this one.

Question 6. (20 points)
We create the data base shown below.
% create the arcs
arc(a,b).
arc(b,a).
arc(a,c).
path1(X,X).
path1(X,Y) :- arc(X,Z), path1(Z,Y).
path2(X,X). path1(X,X).

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path1(X,Y) :- arc(X,Z), path1(Z,Y).
path2(X,X).
path2(X,Y):- path2(X,Z), arc(Z,Y).
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What will be printed out by the queries below? Write yes, no, or stack overflow next to the query.

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?- path1(a,c).
?- path2(a,c).
?- path1(c,b).
?- path2(c,b).
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PS I will add 6 points to the score of the students who took the test. The promisses must be kept ...