COT 3420
SPRING 2006

EXAM # 1

INSTRUCTIONS

1. This test is open book, open notebook.
2. There are 5 questions on the test, for a total of 125 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
4. Circle the answers to questions 1 and 4 on the exam paper. Write the answers to the other questions on blank sheets of paper.
5. If you do not understand the meaning of a question ask me during the test.
6. You have one and a half hour to work on the test.
7. Write your name below.

NAME: _

QUESTIONS

Question 1.(30 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. If 2.3.2 is an address in the domain of the tree t, then $\ldots$must be also an address.

a. 2.3.3
b. 2.3.2.0
c. 2.3
d. 2.4

2. $n[\mathbf{a}, \mathbf{PapaJai}] = \ldots$

a. 3
b. 2

c. 1
d. 0

3. The string ... is not a prefix of **Walid**.
a. λ (the empty string)
b. **W**
c. **Wal**
d. **d**

4. ... is not a binary connective.
a. $\vee$
b. $\neg$
c. $\longrightarrow$

5. $\bigwedge_{i=1}^{3} F_i = \ldots$
a. $((F_1 \wedge F_2) \wedge F_3)$.
b. $(F_1 \wedge (F_2 \wedge F_3))$.
c. $\square$.
d. **T**.

6. The domain of the function $\boxed{\vee}$ is ...
a. The set of formulas $FORM$
b. $FORM \times FORM$
c. $\{0, 1\}$
d. $\{0, 1\} \times \{0, 1\}$

7. The Agreement Theorem tells us that ...
a. every formula belongs to only one category and its main subformulas are unique.
b. the truth value of a formula is determined by the truth values of its atoms.
c. the meaning of a formula does not change when we substitute a subformula by an equivalent subformula.

8. Let F be a formula and let us look at the relation $\mathcal{A}$ and $\mathcal{B}$ agree on F defined on the set of truth assignments. The relation is ...
a. an equivalence.
b. a compatibility relation.

c. a partial order.
d. a total order.

9. The number of non-equivalent formulas with atoms in the set $\{P_1, \ldots, P_n\}$ is ...
a. 2^{2^n}.
b. 4^n.
c. 2^n.
d. n.

10. $n[con, \neg((P_1 \longrightarrow (P_2 \vee P_3)) \wedge \neg P_4)] = \ldots$
a. 2
b. 3
c. 4
d. 5

Question 2. (25 points)

Let $n[atom, F]$ be the number of atom occurrences in the formula F and $n[con, F]$ be the number of binary connective occurrences in F. Prove, by structural induction, that $n[atom, F] > n[con, F]$.

Do not use any theorems or exercises from the book.

Write your answer on a blank sheet of paper.

Question 3. (25 points)

Prove/disprove: If $F \wedge G \wedge H \wedge I$, $G \wedge H \wedge I \wedge J$, $H \wedge I \wedge J \wedge F$, $I \wedge J \wedge F \wedge G$ and $J \wedge F \wedge G \wedge H$ are satisfiable, then $F \wedge G \wedge H \wedge I \wedge J$ is satisfiable.

First you must write Proof or Disproof and then provide the proof or the counter-example.

Write your answer on blank sheets of paper.

Question 4. (30 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. Let F be a satisfiable formula. Then it has ... models.
a. finitely many
b. countably many

c. uncountably many

2. The consequence $F \models \neg F$ is ...
   a. always false.
   b. sometimes true and sometimes false.
   c. always true.

3. Assume that $Con[F] = FORM$, the set of all formulas. ...
   a. Then, F is a tautology.
   b. Then, F is unsatisfiable.
   c. There is no such F.

4. If F and $F \longrightarrow G$ are tautologies, then ...
   a. G is a tautology.
   b. G is satisfiable.
   c. sometimes G is satisfiable and sometimes it is not.

5. If $F \wedge G$ is unsatisfiable, then ...
   a. F is unsatisfiable.
   b. G is unsatisfiable.
   c. $G \equiv \neg F$.
   d. $F \models \neg G$.

6. Let S be a satisfiable set of formulas that has a finite set of models. Then ...
   a. S is infinite.
   b. Every atom must occur in some formula of S.
   c. All but a finite number of atoms must occur in some formula of S.

7. If $\models (F \longleftrightarrow G)$ and F is unsatisfiable, then ...
   a. G is unsatisfiable.
   b. G can be satisfiable.
   c. $G \equiv \neg F$.

8. ...is always true.
   a. $F \models (F \longrightarrow G)$
   b. $F \models (G \longrightarrow F)$
   c. $F \models (G \longrightarrow \neg F)$

9. If $F \models H$ and $G \models H$ then ...
   a. $\models ((F \vee G) \longrightarrow H)$.
   b. $\models (H \longrightarrow (F \vee G))$.
   c. $\models ((F \vee G) \longrightarrow \neg H)$.

10. If $F \equiv G \wedge H$, then ...
   a. $Mod[F] \subseteq Mod[G] \cap Mod[H]$.
   b. $Mod[G] \cap Mod[H] \subseteq Mod[F]$.
   c. $Mod[F] = Mod[G] \cap Mod[H]$.

Question 5. (15 points)

Draw the tree for the formula

$F = \neg(((P_1 \longleftrightarrow \neg P_2) \vee \neg(P_3 \longleftrightarrow \neg P_4)) \wedge ((P_4 \vee \neg P_5) \vee \neg(P_6 \longleftrightarrow \neg P_3)))$

Write your answer on a blank sheet of paper.