COT 3420
SPRING 2006

EXAM # 1 ANSWERS

Question 1.(30 points)
1\. c 2. a 3. d 4. b 5. a 6. d 7. b 8. a 9. a 10. b
Grading Criteria: 3 points for each correct answer.

Question 2. (25 points)
Let $n[atom, F]$ be the number of atom occurrences in the formula F and $n[con, F]$ be the number of binary connective occurrences in F. Prove, by structural induction, that $n[atom, F] > n[con, F]$.
Proof: By structural induction on F.
Case 1: F is an atom. Since atoms are not binary connectives, $n[atom, F] = 1$ and $n[con, F] = 0$. Since $1 > 0$, the inequality $n[atom, F] > n[con, F]$ holds.
Case 2: $F = \neg G$. By induction hypothesis, $n[atom, G] > n[con, G]$. Since the symbol $\neg$ is neither an atom nor a binary connective we have (1) and (2).

(1) $n[atom, F] = n[atom, G]$
and
(2) $n[con, F] = n[con, G]$
Now,
$n[atom, F] = n[atom, G]$ by (1)
$> n[con, G]$ by (IH)
$= n[con, F]$ by (2)
So, $n[atom, F] > n[con, F]$ holds.

Cases 3,4,5,6: $F = (GCH)$ where C is a binary connective. The induction hypothesis gives the inequalities below.

(IH on G) $n[atom, G] > n[con, G]$
(IH on H) $n[atom, H] > n[con, H]$
Since the counts are whole numbers, we can rewrite the last inequality as
(3) $n[atom, H] \geq n[con, H] + 1$
The parentheses are neither atoms nor connectives, so we have (4).
(4) $n[atom, F] = n[atom, G] + n[atom, H]$

The binary connectives of F are the binary connectives of the strings G and H plus C.

(5) $n[con, F] = n[con, G] + n[con, H] + 1$

Now,

$n[atom, F] = n[atom, G] + n[atom, H]$ by (4)

$> n[con, G] + n[atom, H]$ by (IH1)

$\geq n[con, G] + n[con, H] + 1$ by (3)

$= n[con, F]$ by (5)

Again, $n[atom, F] > n[con, F]$ holds. **Q.E.D.**

Grading Criteria:

1. Listing the cases: 3 points.
2. Case 1: 3 points
3. Case 2: 7 points (IH: 2 points, formulas 1 and 2: 2 points, the derivation: 2 points, the reasons: 1 point).
4. Cases 3,4,5, 6: 12 points (IH: 2.5 points, formula 3: 2 points, formulas 4 and 5 : 3 points, derivation 3 points, reasons: 1.5 points)

Question 3. (25 points)

Prove/disprove: If $F \wedge G \wedge H \wedge I$, $G \wedge H \wedge I \wedge J$, $H \wedge I \wedge J \wedge F$, $I \wedge J \wedge F \wedge G$ and $J \wedge F \wedge G \wedge H$ are satisfiable, then $F \wedge G \wedge H \wedge I \wedge J$ is satisfiable.

Disproof: Let $F = P_1 \vee P_2 \vee P_3$, $G = P_1 \vee P_2 \vee \neg P_3$, $H = P_1 \vee \neg P_2 \vee P_3$, $I = P_1 \vee \neg P_2 \vee \neg P_3$, $J = \neg P_1$. The truth tables for the conjunctions $C_1 = F \wedge G \wedge H \wedge I$, $C_2 = G \wedge H \wedge I \wedge J$, $C_3 = H \wedge I \wedge J \wedge F$, $C_4 = I \wedge J \wedge F \wedge G$, $C_5 = J \wedge F \wedge G \wedge H$ and $C = F \wedge G \wedge H \wedge I \wedge J$ are shown below.

The conjunctions C_1, C_2, C_3, C_4, C_5 are satisfiable, but C is not.

P_1	P_2	P_3	F	G	H	I	J	C_1	C_2	C_3	C_4	C_5	C
0	0	0	0	1	1	1	1	0	1	0	0	0	0
0	0	1	1	0	1	1	1	0	0	1	0	0	0
0	1	0	1	1	0	1	1	0	0	0	1	0	0
0	1	1	1	1	1	0	1	0	0	0	0	1	0
1	0	0	1	1	1	1	0	1	0	0	0	0	0
1	0	1	1	1	1	1	0	1	0	0	0	0	0
1	1	0	1	1	1	1	0	1	0	0	0	0	0
1	1	1	1	1	1	1	0	1	0	0	0	0	0

Grading Criteria:

1. Did nor write **Proof** or **Disproof** or wrote both: 0 points.

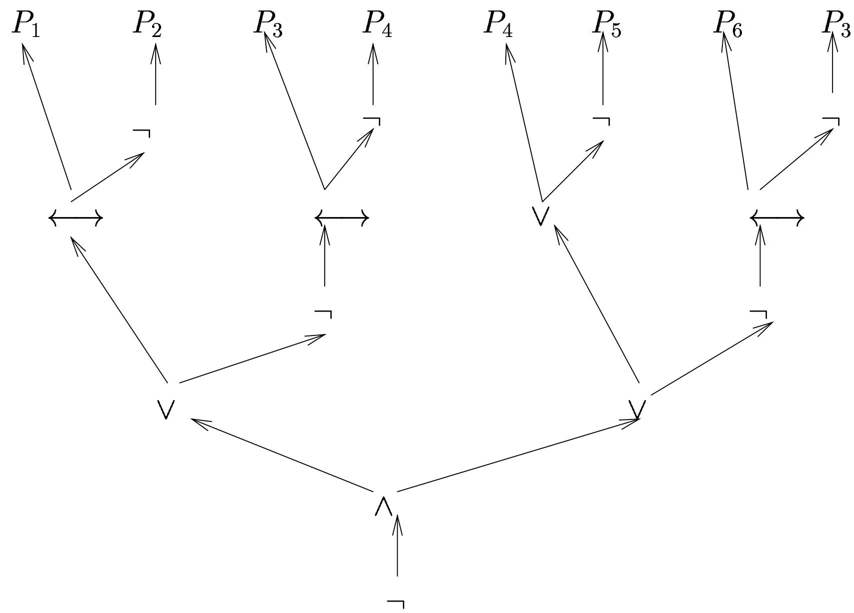

Figure 1: The answer to Q5

2. Wrote **Proof**: 3 points only.
3. Wrote **Disproof**: 8 points.
4. Choosing F,G,H, I, J correctly : 2.5 points each.
5. The truth table: 4.5 points. These points were given to those who chose a correct counter-example.

Question 4. (30 points)

1. c   2. b   3. b   4. a   5. d   6. c   7. a   8. b   9. a   10. c

Grading Criteria: 3 points for each correct answer.

Question 5. (15 points)

Draw the tree for the formula

$F = \neg(((P_1 \longleftrightarrow \neg P_2) \vee \neg(P_3 \longleftrightarrow \neg P_4)) \wedge ((P_4 \vee \neg P_5) \vee \neg(P_6 \longleftrightarrow \neg P_3)))$.

The tree is hown in Figure 1.

Grading Criteria: -2 points for each wrong label or arrow.