COT 3420
Spring 2006

EXAM # 2

INSTRUCTIONS

1. The exam is open book, open notebook.
2. There are 5 questions on the test, for a total of 129 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 2 hours to work on the test.
5. Write the answers to question 1 on the exam paper. Write the other answers on the blank sheets.
6. Print your name below.

NAME: _

QUESTIONS

Question 1. (39 points)

Complete each sentence with the string that not only makes the sentence true, but also states the strongest result.

There is no penalty for wrong guessing, but choose only one answer.

1. If F and G don't have a CNF in common, then ...
   a. $F \equiv G$.
   b. $F \not\equiv G$.
   c. sometimes $F \equiv G$ and sometimes $F \not\equiv G$.

2. Let G and H be two CNF of F. Then, ...
   a. $G = H$.
   b. $G \equiv H$.
   c. $G \not\equiv H$.
   d. sometimes $G \equiv H$ and sometimes $G \not\equiv H$.

3. In the CNF $((((\neg P_1 \vee P_2) \vee \neg P_3) \wedge ((\neg P_4 \vee P_1) \vee P_3)) \wedge (P_2 \vee \neg P_5))$, $L_{2,1} = \ldots$
   a. $\neg P_4$.
   b. P_2.
   c. $((\neg P_4 \vee P_1) \vee P_3)$.

4. …is a CNF of $(A \longleftrightarrow B)$.
   a. $((A \longrightarrow B) \wedge (B \longrightarrow A)$.
   b. $((A \wedge B) \vee (\neg A \wedge \neg B))$.
   c. $((A \vee B) \wedge (\neg A \vee \neg B))$.
   d. $((\neg A \vee B) \wedge (A \vee \neg B))$.

5. Let S be an infinite set of non-equivalent clauses. Then, $Res^*[S]$ …
   a. is finite.
   b. is countably infinite.
   c. is uncountable.
   d. can be finite or countably infinite, depending on S.

6. Let S be an unsatifiable set of non-equivalent formulas and T a subset of S. If T is infinite, then …
   a. T is satisfiable.
   b. T is unsatisfiable.
   c. sometimes T is satisfiable and sometimes T is unsatisfiable.

7. Let S be infinite set of formulas. If every finite subset of S is satisfiable, then …
   a. S is satisfiable.
   b. S is unsatisfiable.
   c. sometimes S is unsatisfiable and sometimes it is not.

8. A set of formulas S is **minimally unsatisfiable** if every proper subset of S (i.e. subsets that are not S) is satisfiable. A minimally unsatisfiable set is …
   a. finite.
   b. infinite.
   c. sometimes S is finite and sometimes it is infinite.

9. Let R_1 and R_2 be two resolvents of clauses C_1 and C_2. If $R_1 \neq R_2$, then ...

a. at least one of C_1, C_2 is a tautology.
b. both C_1, C_2 are tautologies.
c. at least one of R_1, R_2 are a tautologies.
d. both R_1, R_2 are tautologies.

10. The clauses in $Res^{n+1}[S] - Res^n[S]$ have ...
a. S derivation trees of height $\leq n+1$.
b. S derivation trees of height $n+1$.
c. minimal S derivation trees of height $n+1$.

11. Let C and D be two clauses such that $C \subseteq D$. Then, ...
a. $C \models D$.
b. $D \models C$.
c. sometimes, neither $C \models D$ nor $D \models C$ holds.

12. Let S and T be two sets of clauses. If $Res^*[S] = Res^*[T]$, then ...
a. $S = T$.
b. $S \neq T$.
c. $S \equiv T$.
d. $S \not\equiv T$.

13. Let S be an unsatisfiable set of clauses. If S is infinite, then ...
a. there is some $n \in N$ such that $Res^*[S] = Res^n[S]$.
b. for all $n \in N$, $Res^*[S] \neq Res^n[S]$.
c. for some $n \in N$, $\square \in Res^n[S]$.

Question 2. (20 points)

Construct a derivation tree of $\square$ from
$\{\{A, B, C\}, \{A, B, \neg C\}, \{\neg B, D\}, \{\neg B, \neg D, E\}, \{\neg B, \neg D, \neg E\}, \{\neg A, F, G\}, \{\neg A, \neg F\}, \{\neg A, F, \neg G\}\}$.
Draw the tree on a blank sheet of paper.

Question 3. (25 points)

Prove the generalized De Morgan's law,
$F \vee \bigwedge_{i=1}^{n} G_i \equiv \bigwedge_{i=1}^{n} (F \vee G_i)$.
Write your proof on a blank sheet of paper.

Hint: Use induction on n.

Question 4. (25 points)

Show, by structural induction, that the set $S = \{F \wedge \neg G, \mathbf{T}\}$ is adequate where $\mathbf{T}$ is a tautology.

Write your proof on a blank sheet of paper.

Question 5. (20 points)

Find a CNF for the formula

$F = [\neg(A \wedge (B \vee C)) \longleftrightarrow (\neg B \vee \neg(C \vee D))]$.

Show your work on a white sheet of paper.