COT 3420
SPRING 2006

EXAM # 3

INSTRUCTIONS

1. The exam is open book, open notebook.
2. There are 4 questions on the test, for a total of 80 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour to work on the test.
5. Write the answers to question 1 on the exam paper. Write the other answers on the blank sheets.
6. Print your name below.

NAME: _____

QUESTIONS

Question 1. (30 points)

The universe of structure $\mathcal{A}$ is $\{3, 4, 5\}$. The $\mathcal{A}$ interpretations of a, x, and y are $a^{\mathcal{A}} = 4$, $x^{\mathcal{A}} = 5$, $y^{\mathcal{A}} = 3$. The tables for the functions $f^{\mathcal{A}}$ and $g^{\mathcal{A}}$ and the predicates $P^{\mathcal{A}}$ and $Q^{\mathcal{A}}$ are shown in Figure 1. Evaluate the terms and formulas below. Do not show your work, just write the answer after the equal sign.

1. $\mathcal{A}[f(x)] =$
2. $\mathcal{A}[g(x, x)] =$
3. $\mathcal{A}[g(g(a, y), f(x))] =$
4. $\mathcal{A}[Q(g(y, x))] =$
5. $\mathcal{A}[P(x, f(a))] =$
6. $\mathcal{A}[E(f(x), g(y, a))] =$
7. $\mathcal{A}[\forall x P(f(x), x)] =$
8. $\mathcal{A}[\forall x Q(f(x))] =$

x	$f^{\mathcal{A}}[x]$
3	5
4	4
5	5

$f^{\mathcal{A}}$

x \ y	3	4	5
3	5	3	4
4	3	4	5
5	4	5	3

$g^{\mathcal{A}}$

x \ y	3	4	5
3	0	0	0
4	0	1	1
5	1	0	1

$P^{\mathcal{A}}$

x	$Q^{\mathcal{A}}[x]$
3	1
4	1
5	0

$Q^{\mathcal{A}}$

Figure 1: Tables for Question 1

9. $\mathcal{A}[\exists x \forall y \neg P(x, y)] =$
10. $\mathcal{A}[\forall x \exists y P(y, x)] =$

Question 2. (20 points)

Prove that if x is not free in G, then $\exists x F \longrightarrow G \equiv \forall x (F \longrightarrow G)$.

Write the proof on a blank sheet of paper.

Question 3. (25 points)

We define the set of connectives $S = \{F \longrightarrow G, \Box, \exists x F\}$ where x can be any variable. Prove, by structural induction on F, that S is adequate for FOL.

Write the proof of a blank sheet of paper.

Question 4. (10 points)

Rectify the formula $F = \forall x\{\forall y[(P(x, y) \vee \exists z Q(z, y)) \wedge \exists y \neg P(y, z)] \wedge \forall z[\exists x(\neg P(z, x) \vee Q(y, z)) \wedge \forall z(P(x, z) \vee Q(z, y))]\}$.

Show your work on a white sheet of paper.