COT 3420
Spring 2006

FINAL EXAM

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 10 questions on the test, for a total of 135 points.
3. For the multiple choice question, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 2 hours and a half to complete the exam.
5. Mark the answers to questions 2,3,4,5,6, and 9 on the exam paper. Write the answers to the other questions on the blank sheets.
6. No talking to each other during the test!
7. Write your name below.

NAME: _

QUESTIONS

Question 1. (15 points)

Write $E(F,2)$ for $F = \forall x \forall y(\neg P(x,a) \wedge (P(f(x),y) \vee P(x,f(y))))$. Write your answer on a blank sheet of paper.

Question 2. (10 points)

Rectify the formula $F = \forall x[\exists y(P(x,y) \vee Q(x,z)) \wedge (\exists z \neg Q(x,z) \vee \neg Q(x,y))] \wedge \forall y[\exists x(P(x,y) \vee \neg P(a,x)) \wedge \forall y(\neg P(y,x) \vee Q(z,y))]$. Write your answer below.

Question 3. (5 points)

Close the formula $F = \forall y \forall u F^M[x,y,z,u,v,w]$ where F^M, the matrix of F, has free occurrences of x, y, z, u, v, w.

Write your answer below.

Question 4. (10 points)

Skolemize the formula $F = \exists x_1 \exists x_2 \forall x_3 \exists x_4 \exists x_5 \forall x_6 \exists x_7 F^M$, where F^M, the matrix of F, contains the constant a and the function symbols f and g.

Write your answer below.

Question 5. (10 points)

Find a prenex form for

$F = \neg\{\forall x[\exists y(P(x,y) \vee Q(x,y)) \wedge \exists z(\neg P(z,x) \vee Q(x,z))] \wedge \exists u[\forall v(P(v,u) \vee \neg Q(u,a)) \wedge \forall w(\neg P(w,u) \vee Q(b,w))]\}$.

Write your answer below.

Question 6. (10 points)

Find a mgu for $S = \{P(x_1, f(x_2), g(x_1, x_3)), P(h(x_2), x_4, g(h(x_5), f(x_5))), P(x_6, f(h(x_7)), x_8)\}$ or prove that the set is not unifiable.

Question 7. (20 points)

Prove by first order resolution that $S = \{\{P(x,y), P(f(y),z), Q(x,y)\}, \{\neg P(y,a), Q(y,x)\}, \{R(h(x),x), \neg Q(x,a)\}, \{\neg R(x,y), \neg R(h(z),y), \neg Q(z,u)\}\}$ is unsatisfiable.

For each resolution step specify the relabelings, the unification set and the mgu. Do the minimal number of unifications. Draw your tree on a blank sheet of paper.

Question 8. (20 points)

We define the set of connectives $S = \{\downarrow, \forall x\}$ where $F \downarrow G = \neg(F \vee G)$. Prove, by structural induction on F, that S is adequate for FOL.

Write your proof on a blank sheet of paper.

Question 9. (15 points)

Display a formula F that is satisfied if and only if the structure has 4 elements.

Write the formula below.

Question 10. (20 points)

Let u and v two variables that do not occur in F, neither free nor as arguments to quantifiers. Prove that $\forall x \forall y F \models \forall u \forall v F[x/f(u,v), y/g(u)]$. Write your answer on a blank sheet of paper.

Hint: Use the Translation Lemma.