COT 3420
SUMMER A 2003
Section 1

EXAM # 1 ANSWERS

Question 1.(20 points)
1. d 2. c 3. b 4. b 5. b 6. c 7. c 8. a 9. a 10. c

Question 2. (20 points)
The proof is by structural induction on F.
Case 1: F is an atom. Then $F = P_i$ for some $i \in N$. So, $n[atom, P_i] = 1$, and $n[con, P_i] = 0$. The equation $n[atom, F] = n[con, P_i] + 1$ becomes
$1 = 0 + 1$
which is true.
Case 2: $F = \neg G$. We assume that the equation holds for G.
(IH) $n[atom, G] = n[con, G] + 1$
Now we relate the F counts to the G counts.
(1) $n[atom, F] = n[atom, G]$ because $F = \neg G$ and $\neg$ is not an atom
(2) $n[con, F] = n[con, G]$ because $F = \neg G$ and $\neg$ is not a binary connective
Now we compute $n[atom, F]$.
$n[atom, F] = n[atom, G]$ by (1)
$= n[con, G] + 1$ by (IH)
$= n[con, F] + 1$ by (2)
So, $n[atom, F] = n[con, F] + 1$.
Cases 3, 4, 5, 6: $F = (GCH)$. We assume that the proposition is true for G and H.
(IH1) $n[atom, G] = n[con, G] + 1$
(IH2) $n[atom, F] = n[con, F] + 1$
We express the F counts as functions of G and H counts. Since $F = (GCH)$ and the symbols (, C,) are not atoms,
(3) $n[atom, F] = n[atom, G] + n[atom, H]$.
The connectives of F are the connectives of G and H plus C.
(4) $n[con, F] = n[con, G] + 1 + n[con, H]$
Now we relate $n[atom, F]$ to $n[con, F]$.

$n[atom, F] = n[atom, G] + n[atom, H]$ by (3)
$= n[con, G] + 1 + n[atom, H]$ by (IH1)
$= n[con, G] + 1 + n[con, H] + 1$ by (IH2)
$= (n[con, G] + 1 + n[con, H]) + 1$ by grouping
$= n[con, F] + 1$ by (4)
So, $n[atom, F] = n[con, F] + 1$.

Grading Criteria: 1. Listing the 6 cases: 2 points.

2. Case 1: 2 points.

3. Case 2: 7 points; The IH - 1 point, the 2 count equations - 2 points, the derivation - 3 points, the reasons - 1 point.

4. Cases 3,4,5,6: 9 points; IH's - 2points, the count equations - 2 points, the derivation - 4 points, the reasons - 1 point.

Question 3. (10 points)

The tree is shown in Figure 1.

Grading Criteria: 1 point off for each out of order node.

Question 4. (15 points)

The proof is by mathematical induction on n.

Basis: $n = 0$. Then $\bigvee_{i=0}^{0} P_i = P_0$, a symbol. So, $|\bigvee_{i=0}^{0} P_i| = 1$, and the equation $|\bigvee_{i=0}^{n} P_i| = 4n + 1$ becomes

$1 = 4 * 0 + 1$

which is true.

Inductive Step: Assume that the equation is true for n.

(IH) $|\bigvee_{i=0}^{n} P_i| = 4n + 1$

Let us prove

$|\bigvee_{i=0}^{n+1} P_i| = 4(n + 1) + 1$

The know that $\bigvee_{i=0}^{n+1} P_i$ denotes the string $(\bigvee_{i=0}^{n} P_i \vee P_{n+1})$.

So, the length of $\bigvee_{i=0}^{n+1} P_i$ is the length of $\bigvee_{i=0}^{n} P_i$ plus the lengths of the symbols (, $\vee$, P_{n+1} and). So,

(1) $|\bigvee_{i=0}^{n+1} P_i| = |\bigvee_{i=0}^{n} P_i| + 4$

Now we relate $|\bigvee_{i=0}^{n+1} P_i|$ to n.

$|\bigvee_{i=0}^{n+1} P_i| = |\bigvee_{i=0}^{n} P_i| + 4$ by (1)
$= (4n + 1) + 4$ by (IH)
$= (4n + 4) + 1$ by grouping
$= 4(n + 1) + 1$ by distributivity

So, we proved that $|\bigvee_{i=0}^{n+1} P_i| = 4(n + 1) + 1$.

Grading Criteria: 1. Listing the 2 cases: 2 points.

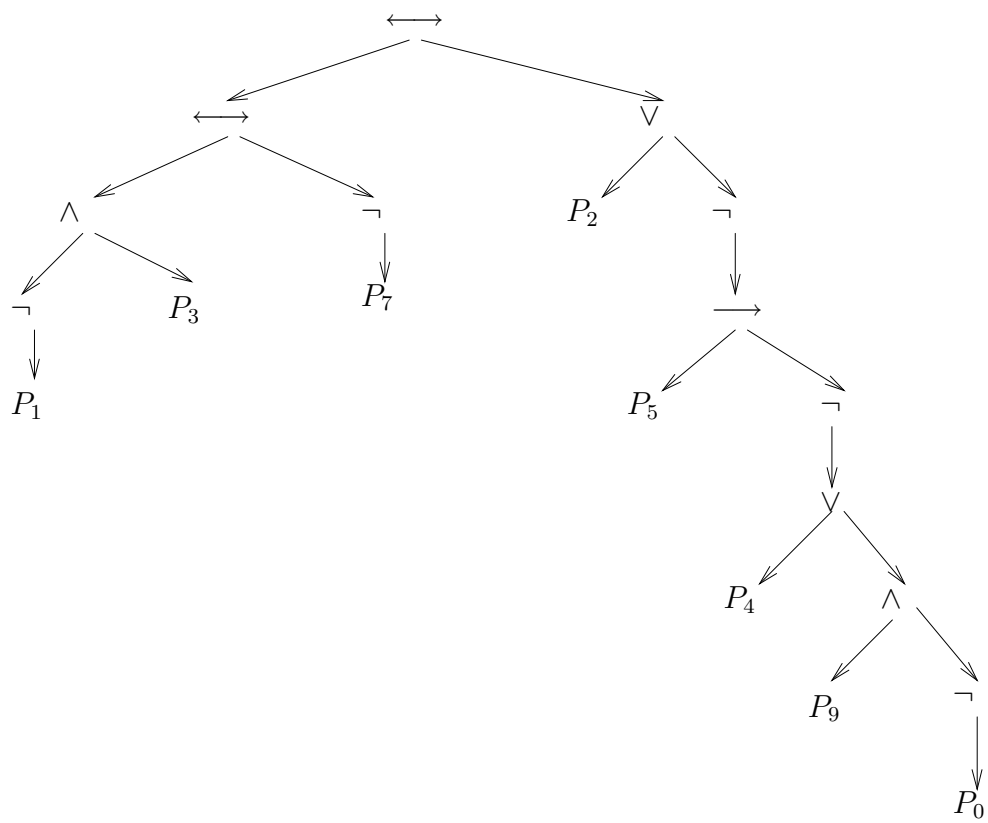

Figure 1: The answer for Question 3

2. Basis: 3 points.

3. Inductive Step: 10 points. The IH - 1 point; the formula (1) - 5 points, the derivation + reasons=4 points.

Question 5. (10 points)

$S = \{P_1, P_2, P_3, P_0, P_4, P_7, \neg P_2, \neg P_3, (P_4 \longleftrightarrow P_7), (P_1 \vee \neg P_2), (P_2 \wedge \neg P_3), (P_0 \wedge (P_4 \longleftrightarrow P_7)), ((P_1 \vee \neg P_2) \longrightarrow (P_2 \wedge \neg P_3)), \neg(P_0 \wedge (P_4 \longleftrightarrow P_7)), \neg((P_1 \vee \neg P_2) \longrightarrow (P_2 \wedge \neg P_3)), (\neg((P_1 \vee \neg P_2) \longrightarrow (P_2 \wedge \neg P_3)) \longleftrightarrow \neg(P_0 \wedge (P_4 \longleftrightarrow P_7)))\}$

Grading Criteria: $\frac{10}{16}$(number-of-right-answers - number-of-wrong-answers)