COT 3420
SUMMER A 2003
Section 2

EXAM # 1

INSTRUCTIONS

1. The exam is open book, open notebook.
2. There are 5 questions on the test, for a total of 75 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
4. If you do not understand the meaning of a question ask me during the test.
5. You have 1 hour to work on the test.
6. Write all your answers on the exam sheet.
7. Write your name below.

NAME: _

QUESTIONS

Question 1.(20 points)

For each of the following relations select the string that provides the most accurate description. There is no penalty for wrong guessing, but choose only one answer.

1. If 1.1.1 is in the domain of the tree t, then ... must also be in the domain.
   a. 1.1
   b. 0.1
   c. 1.1.1.1
   d. 1.1.2

2. The string ... is a formula.
   a. $P \vee P_2$
   b. $(P_2 \longrightarrow P_0)$
   c. $(P_0 \longleftrightarrow P_{-1})$

d. $(\neg P_3)$

3. A formula cannot end with ...
a.))
b. P_2
c. $\neg$
d. $\neg P_3$

4. Let u be a prefix of v. Then ... is a prefix of v.
a. a prefix of u
b. a suffix of u
c. a suffix of v
d. a substring of u

5. Let X and Y be two non-empty strings such that $F = XY$ is a formula. Then ...
a. $n[\neg, X] \geq 1$.
b. $n[), Y] \geq 1$.
c. $n[\neg, X] + n[), Y] \geq 1$.
d. $n[(, X] = n[), Y]$.

6. $n[a, babaca] =$...
a. 0
b. 1
c. 2
d. 3

7. The second occurrence of 3 in 12343 occurs at index ...
a. 2
b. 3
c. 4
d. 5

8. Let F, G, H, I, J be 5 formulas such that $((F \vee G) \wedge H) = (I \wedge J)$. Then ...
a. I is a prefix of $(F \vee G)$.
b. $(F \vee G)$ is a prefix of I.
c. H is a suffix of J.

d. $I = (F \vee G)$.

9. $\bigwedge_{i=4}^{2} F_i$ represents ...
   a. $\Box$.
   b. $\mathbf{T}$.
   c. $(F_2 \wedge (F_3 \wedge F_4)))$.
   d. $((F_2 \wedge F_3) \wedge F_4)$.

10. Let t be a formula tree and A an address of t. We write t/A for the subtree of t with root A. Let us assume that $convert[t]$ outputs the formula $F = (G \vee H)$, where G,H are formulas. Then ...
   a. $convert[t/1] = H)$.
   b. $convert[t/1] = (G$.
   c. $convert[t/1] = \vee H$.
   d. $convert[t/1] = H$.

Question 2. (20 points)

Prove by structural induction that for every suffix S of F, $n[), S] \geq n[con, S] \geq n[(, S]$. The count $n[con, S]$ is the number of binary connectives in the string S. Write your proof below and on the opposite page.

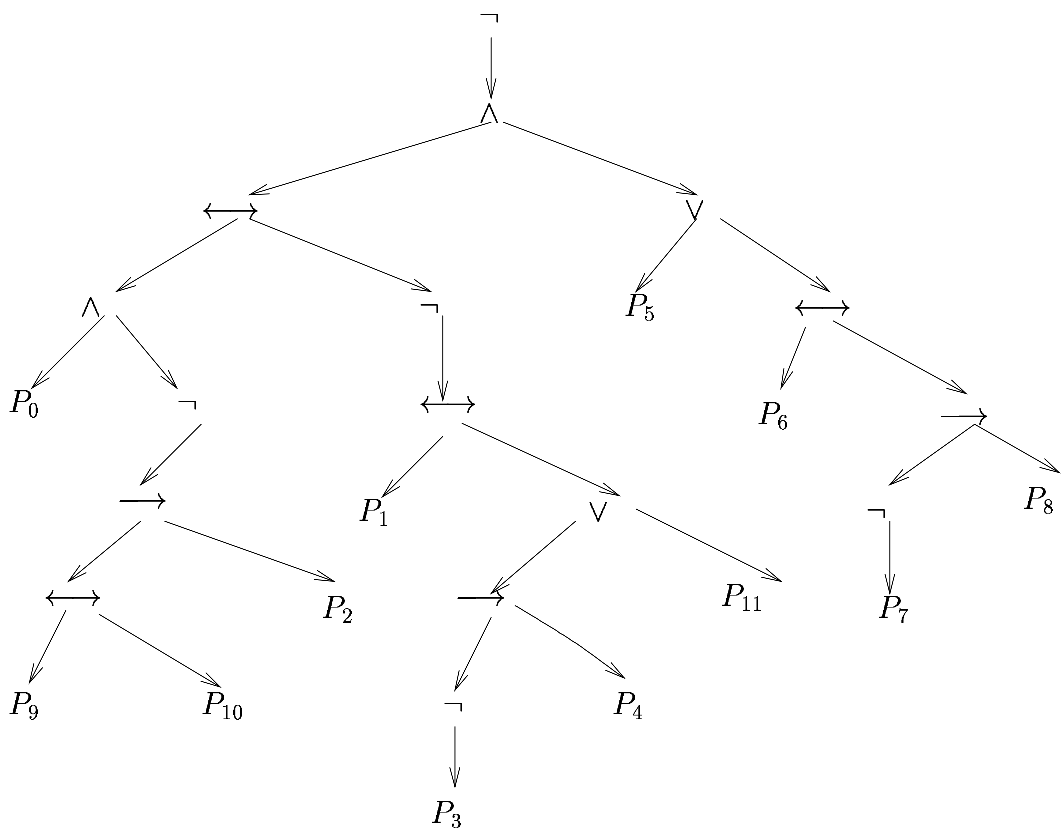

Figure 1: The tree for Question 3

Question 3. (10 points)

Find the formula that corresponds to the tree from Figure 1.
Write your answer below.

Question 4. (15 points)

Prove by mathematical induction that $n^3 - 16n$ is divisible by 3. Write your answer on the opposite page.

Question 5. (10 points)

Find all subformulas of $F = \neg(((P_1 \vee P_2) \longrightarrow \neg P_3) \longleftrightarrow ((P_1 \wedge \neg P_3) \longrightarrow (\neg P_1 \vee \neg(P_0 \wedge \neg P_4))))$.

Write your answer below.