COT 3420
Section U2
FALL 2006

EXAM # 1

INSTRUCTIONS

1. This test is open book, open notebook.
2. There are 5 questions on the test, for a total of 100 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
4. Circle the answers to question 1 on the exam paper. Write the answers to the other questions on blank sheets of paper.
5. If you do not understand the meaning of a question ask me during the test.
6. You have 1 hour to work on the test.
7. Write your name below.

NAME: _

QUESTIONS

Question 1.(28 points)

For each of the following 14 statements select the string that not only makes the assertion true, but also states the strongest possible result. There is no penalty for wrong guessing, but choose only one answer.

1. If F is unsatisfiable, then $\neg F$ is ...
   a. a tautology.
   b. satisfiable, but not a tautology.
   c. unsatisfiable.

2. If F, G are satisfiable, then $F \longleftrightarrow G$ is ...
   a. satisfiable.
   b. satisfiable or unsatisfiable, depending on F and G.
   c. unsatisfiable.

3. If both $F \vee G$ and $\neg G$ are satisfiable, then ...
   a. F must be satisfiable.
   b. F must be unsatisfiable.
   c. F can be satisfiable or unsatisfiable.

4. If $F \models G$ and $F \models \neg G$, then ...
   a. F must be unsatisfiable.
   b. F must be a tautology.
   c. F can be satisfiable or unsatisfiable.

5. $n[con, (((P_0 \vee \neg P_1) \wedge \neg(P_2 \longrightarrow P_3)) \wedge (P_4 \vee P_5))] = \ldots$
   a. 2.
   b. 5.
   c. 7.

6. ... is not a prefix of **AliBaba**.
   a. λ
   b. **AliBaba**
   c. **Ali**
   d. **Baba**

7. In the CNF form $F = ((((P_0 \vee P_1) \wedge ((P_2 \vee P_3) \vee P_3)) \wedge P_4) \wedge (P_5 \vee P_6))$, $L_{4,1}$ ...
   a. does not exits.
   b. P_4.
   c. P_5.
   d. P_6.

8. A clause is *positive* if is not empty and all its literals are atoms. Let S be a set of clauses that does not contain $\Box$. If S has no positive clauses, then, ...
   a. S is satisfiable.
   b. S is unsatisfiable.
   c. we cannot tell whether S is satisfiable or not.

9. Let S be a set of clauses and C and D be two clauses such that $C \subseteq D$. If $S \cup \{D\}$ is unsatisfiable, then ...

a. S is unsatisfiable.
b. $S \cup \{C\}$ is satisfiable.
c. $S \cup \{C\}$ is unsatisfiable.
d. $S \cup \{C\}$ can be satisfiable or unsatisfiable.

10. If 2.0.1 is an address in a tree, then ... must also be an address in the same tree.
a. 1
b. 1.0
c. 2.0.2
d. 2.1

11. If the formulas F and G have a common CNF, then ...
a. $F = G$.
b. $F \equiv G$.
c. sometimes $F \equiv G$ and other times $F \not\equiv G$.

12. A set of formulas S is called minimally unsatisfiable iff
1. S is unsatisfiable, and
2. for all subsets T of S, $T \neq S$ implies that T is satisfiable.
Then, the minimally unsatisfiable sets ...
a. are finite.
b. can be countably infinite.
c. can be uncountable.

13. If F is a tautology, then $Con[\{F\}]$ ...
a. is empty.
b. is the set of all tautologies.
c. is the set of all formulas.

14. The statement *All infinite sets of formulas are equivalent to sets of clauses* is ... true.
a. always
b. sometimes, but not always
c. never

Question 2. (15 points)

Let F, G, H be 3 formulas. We define the string $H[F/G]$ by the rules below.

1. If the string F does not occur in H, then $H[F/G] = H$.

2. If the string F occurs in H, $H[F/G]$ is the string obtained by replacing an occurrence of F by G.

Prove, by structural induction on H, that $H[F/G]$ is a formula.

Write your answer on a blank sheet of paper.

Question 3. (25 points)

Proof or disproof: If the formulas $F \wedge G \wedge H \wedge I \wedge J$, $G \wedge H \wedge I \wedge J \wedge K$, $H \wedge I \wedge J \wedge K \wedge F$, $I \wedge J \wedge K \wedge F \wedge G$, $J \wedge K \wedge F \wedge G \wedge H$, $K \wedge F \wedge G \wedge H \wedge I$ are satisfiable, then $F \wedge G \wedge H \wedge I \wedge J \wedge K$ is also satisfiable.

First you must write if you go for the proof or for disproof. Then, write the proof or display the counterexample on a blank sheet of paper.

Question 4. (14 points)

Construct a derivation tree of $\square$ from
$S = \{\{A, B, C\}, \{A, \neg B\}, \{\neg C, D\}, \{\neg C, \neg D\},$
$\{\neg F, \neg G\}, \{\neg F, G\}, \{\neg A, F\}\}$.
Draw your tree on a blank sheet of paper.

Question 5. (19 points)

Apply the algorithm given in the book to find a CNF for
$F = \neg[((A \vee B) \wedge C) \longleftrightarrow \neg(B \vee D)]$.
Show your work on a white sheet of paper.