COT 3420
FALL 2006
Section U1

EXAM # 2

INSTRUCTIONS

1. The exam is open book, open notebook.
2. There are 7 questions on the test, for a total of 100 points.
3. For Question 1, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour and 15 minutes to work on the test.
5. Write the answers to questions 1, 3, 4, 5, and 6 on the exam paper. Write the other answers on the blank sheets.
6. Print your name below.

NAME: _

QUESTIONS

Question 1. (27 points)

The universe of structure $\mathcal{A}$ is $\{3, 4, 5, 6\}$. The $\mathcal{A}$ interpretations of a, x, and y are $a^{\mathcal{A}} = 3$, $x^{\mathcal{A}} = 5$, $y^{\mathcal{A}} = 6$. The tables for the functions $f^{\mathcal{A}}$ and $g^{\mathcal{A}}$ and the predicate $P^{\mathcal{A}}$ are shown in Figure 1. Evaluate the terms and formulas below. Do not show your work, just write the answer after the equal sign.

1. $\mathcal{A}[f(x)] =$

2. $\mathcal{A}[g(x, y)] =$

3. $\mathcal{A}[g(g(a, y), f(x))] =$

4. $\mathcal{A}[P(x, f(a))] =$

u	$f^{\mathcal{A}}[u]$
3	3
4	5
5	3
6	5

$f^{\mathcal{A}}[u]$

$u \backslash v$	3	4	5	6
3	3	3	3	3
4	3	4	5	6
5	3	5	3	5
6	3	6	5	4

$g^{\mathcal{A}}[u,v]$

$u \backslash v$	3	4	5	6
3	1	0	1	0
4	0	0	0	0
5	0	1	0	1
6	0	0	0	0

$P^{\mathcal{A}}[u,v]$

Figure 1: Tables for Question 1

5. $\mathcal{A}[E(f(y), g(x,a))] =$

6. $\mathcal{A}[\forall x P(x, f(x))] =$

7. $\mathcal{A}[\forall y P(f(y), y)] =$

8. $\mathcal{A}[\forall x \exists y P(y,x)] =$

9. $\mathcal{A}[\exists x \forall y \neg P(x,y)] =$

Question 2. (20 points)

Show that the set of connectives $S = \{\exists x \neg F \wedge G\}$ is adequate for FOL. Hint: Let $\sigma[x,F,G] = \exists x \neg F \wedge G$. Find 3 formulas $\phi_{\neg}[F]$, $\phi_{\wedge}[F,G]$ and $\phi_{\exists x}[F]$ that contain only the σ-operator, the meta-variable(s) specified in the brackets, and atomic formulas, such that

$\phi_{\neg}[F] \equiv \neg F$, $\phi_{\wedge}[F,G] \equiv (F \wedge G)$ and $\phi_{\exists x}[F] \equiv \exists x F$,

and use the fact that $T = \{\neg F, F \wedge G, \exists x F\}$ is adequate.

Write your answer on a blank sheet of paper.

Question 3 (10 points)

Rectify the formula

$F = [\forall x \neg P(x,x) \wedge \forall x \forall y \exists y P(x,y)] \wedge [\forall x \forall y \forall z ((\neg P(x,y) \vee \neg P(y,z)) \vee P(x,z)) \wedge P(y,y)]$

Write your answer below.

Question 4 (8 points)

Skolemize the formula $F = \exists x \forall y \exists z \forall u \forall v \exists w F^M$, where F^M is the matrix of F. The matrix of F contains the constants a and b and the function symbols f and g. Don't show your work, just write the Skolemized formula.

Write the substitutions as [variable/term]. Display the answer below.

Question 5 (10 points)

Find a prenex form for

$F = [\forall x(\exists y P(x,y) \vee \forall z Q(x,z)) \wedge \exists u(\forall v \neg P(u,v) \vee \exists w \neg Q(w,u))]$.

Don't show your work, just write the prenex form. Display your answer below.

Question 6 (5 points)

Close the formula $F = \forall z \exists v P(x,y,z,u,v)$.

Write your answer below.

Question 7 (20 points)

Let F be a rectified formula and u, v be two variables that do not occur in F. Prove that $\forall x \forall y F \models \forall u \forall v F[x/f(u), y/g(v)]$. Here, f and g are unary functions, and x, y, u, v are 4 different variables. Write your answer on a blank sheet of paper.

Hint: Apply the translation lemma.