COT 3420
FALL 2006
Section U2

EXAM # 2

INSTRUCTIONS

1. The exam is open book, open notebook.
2. There are 7 questions on the test, for a total of 100 points.
3. For Question 1, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour and 15 minutes to work on the test.
5. Write the answers to questions 1, 3, 4, 5, and 6 on the exam paper. Write the other answers on the blank sheets.
6. Print your name below.

NAME: _

QUESTIONS

Question 1. (27 points)

The universe of structure $\mathcal{A}$ is $\{3, 4, 5, 6\}$. The $\mathcal{A}$ interpretations of a, x, and y are $a^{\mathcal{A}} = 5$, $x^{\mathcal{A}} = 6$, $y^{\mathcal{A}} = 3$. The tables for the functions $f^{\mathcal{A}}$ and $g^{\mathcal{A}}$ and the predicate $P^{\mathcal{A}}$ are shown in Figure 1. Evaluate the terms and formulas below. Do not show your work, just write the answer after the equal sign.

1. $\mathcal{A}[f(x)] =$

2. $\mathcal{A}[g(x, y)] =$

3. $\mathcal{A}[g(g(a, y), f(x))] =$

4. $\mathcal{A}[P(x, f(a))] =$

u	$f^{\mathcal{A}}[u]$
3	4
4	4
5	6
6	5

$f^{\mathcal{A}}[u]$

$u \backslash v$	3	4	5	6
3	3	3	3	3
4	3	4	5	6
5	3	5	3	5
6	3	6	5	4

$g^{\mathcal{A}}[u, v]$

$u \backslash v$	3	4	5	6
3	0	0	0	0
4	1	1	0	0
5	0	0	0	1
6	0	0	1	1

$P^{\mathcal{A}}[u, v]$

Figure 1: Tables for Question 1

5. $\mathcal{A}[E(f(y), g(x, a))] =$

6. $\mathcal{A}[\forall x P(f(x), x)] =$

7. $\mathcal{A}[\forall y P(y, f(y))] =$

8. $\mathcal{A}[\forall x \exists y P(y, x)] =$

9. $\mathcal{A}[\forall x \exists y P(x, y)] =$

Question 2. (15 points)

Show that the set of connectives $S = \{\forall x \neg F \vee \exists y \neg G\}$ is adequate for FOL.

Hint: Let $\sigma[x, F, y, G] = \forall x \neg F \vee \exists y \neg G$. Find 3 formulas $\phi_{\neg}[F]$, $\phi_{\vee}[F, G]$ and $\phi_{\forall x}[F]$ that contain only the σ-operator, the meta-variable(s) specified in the brackets, and atomic formulas, such that

$\phi_{\neg}[F] \equiv \neg F$, $\phi_{\vee}[F, G] \equiv (F \vee G)$ and $\phi_{\forall x}[F] \equiv \forall x F$,

and use the fact that $T = \{\neg F, F \vee G, \forall x F\}$ is adequate.

Write your answer on a blank sheet of paper.

Question 3 (10 points)

Rectify the formula

$F = \forall x[\forall y(P(x, y) \vee Q(x, z)) \wedge \exists z \exists x \forall y(\neg P(x, y) \vee \forall z \neg Q(x, z))]$.

Write your answer below.

Question 4 (8 points)

Skolemize the formula $F = \exists x \exists y \forall z \exists u \forall v \exists w F^M$, where F^M is the matrix of F. The matrix of F contains the constants a and b and the function symbols f and g. Don't show your work, just write the Skolemized formula.

Write the substitutions as [variable/term]. Display the answer below.

Question 5 (10 points)

Find a prenex form for

$F = \forall x[\exists y P(x, y) \vee \forall z Q(x, z)] \wedge \exists u[\forall v \neg P(u, v) \wedge \exists w \neg Q(w, u)]$.

Don't show your work, just write the prenex form. Display your answer below.

Question 6 (5 points)

Close the formula $F = \forall z \exists v P(x, y, z, u, v)$.

Write your answer below.

Question 7 (20 points)

a. (10 points)

Let x be free for y in F. Then, prove that $\forall x \forall y F \models \forall x F[y/x]$.

Hint: Apply the translation lemma.

b. (10 points)

Show that the consequence does not always hold when x is not free for y in F.

Write your answer on a blank sheet of paper.