COT 3420
Section U1
SPRING 2007

EXAM # 1

INSTRUCTIONS

1. This test is open book, open notebook.
2. There are 5 questions on the test, for a total of 100 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
4. Circle the answers to question 1 on the exam paper. Write the answers to the other questions on blank sheets of paper.
5. If you do not understand the meaning of a question ask me during the test.
6. You have 1 hour to work on the test.
7. If you took the induction test you get 5 extra points.
8. Write your name below.

NAME: _

QUESTIONS

Question 1. (28 points)

For each of the following 14 statements select the string that not only makes the assertion true, but also states the strongest possible result. There is no penalty for wrong guessing, but choose only one answer.

1. If 1.2 is an address in the tree, then ... is also an address in the same tree.
   a. 1.3
   b. 1.2.0
   c. 2
   d. 1.1

2. $n[con, (\neg((P_0 \longrightarrow P_1) \wedge P_2) \longleftrightarrow (P_3 \wedge P_4))] = \ldots$

a. 1
b. 3
c. 4
d. 5

3. ...is always true.
a. $F, \neg F \models G$
b. $\neg F, \neg G \models F$
c. $\neg F, G \models F$
4. $F, G \models \neg F$

4. Let $S = \{(\neg F \vee \neg G)\}$ be a set of connectives. Then ...is an S-formula.
a. $((\neg(P_0 \vee P_1) \vee \neg P_2)$
b. $(\neg P_0 \wedge \neg(P_1 \vee \neg P_2))$
c. $(\neg P_0 \vee \neg(\neg P_1 \vee \neg P_2))$
d. $(\neg(\neg P_0 \vee \neg P_1) \vee \neg\neg P_1)$

5. In the CNF $F = (((P_0 \wedge P_1) \wedge (((P_2 \vee P_3) \vee P_4) \vee P_5)) \wedge (((P_6 \vee P_7) \vee P_8) \vee P_9))$, $L_{4,3}$...
a. $= P_4$.
b. $= P_5$.
c. $= P_8$.
d. $= P_9$.
e. does not exist.

6. If $F \wedge G$ is satisfiable, then ...
a. F is satisfiable.
b. F is unsatisfiable.
c. sometimes F is satisfiable and sometimes it is not.

7. If $F \models (\neg G \vee H)$ then ...
a. $F, \neg G \models H$.
b. $F, G \models H$.
c. $\neg F, \neg G \models H$.

8. The set of models of F cannot be ...
a. finite.
b. countably infinite.

c. uncountable.

9. The domain of $\boxed{\wedge}$ is ...
a. $FORM$.
b. $\{0,1\}$.
c. $FORM \times FORM$.
d. $\{0,1\} \times \{0,1\}$.

10. If $F \equiv (G \vee H)$, then ...
a. $Mod[F] \subseteq (Mod[G] \cup Mod[H])$.
b. $Mod[F] = (Mod[G] \cup Mod[H])$.
c. $(Mod[G] \cup Mod[H]) \subseteq Mod[F]$.
d. for some F, G, H, neither $Mod[F] \subseteq (Mod[G] \cup Mod[H])$ nor $(Mod[G] \cup Mod[H]) \subseteq Mod[F]$ holds.

11. The formulas F and G have no common CNF's. Then ...
a. $F \equiv G$.
b. $F \not\equiv G$.
c. sometimes $F \equiv G$, other times, $F \not\equiv G$.

12. $\bigwedge_{i=2}^{4} F_i = \ldots$
a. $((F_2 \wedge F_3) \wedge F_4)$.
b. $(F_2 \wedge (F_3 \wedge F_4))$.
c. $\Box$.
d. $\mathbf{T}$.

13. Let $S = \{C_0, C_1, \ldots, C_n, \ldots\}$ be an infinite set of clauses. If S is unsatisfiable, then ...
a. for some $n \in N$, $\bigwedge_{i=0}^{n} C_i$ is unsatisfiable.
b. for infinitely many $n \in N$, $\bigwedge_{i=0}^{n} C_i$ is satisfiable.
c. $Res[S] = S$.

14. If S is a set of clauses, then ...
a. $Res^*[Res^*[S]] \subseteq Res^*[S]$.
b. $Res^*[S] \subseteq Res^*[Res^*[S]]$.
c. $Res^*[Res^*[S]] = Res^*[S]$.
d. $Res^*[Res^*[S]] \equiv Res^*[S]$.

Question 2. (25 points)

Do parts a and b.

a. (13 points) Prove that the set of connectives $S_1 = \{F \vee \neg G\}$ is not adequate.

b. (12 points) Show that the set of connectives $S_2 = \{(F \wedge \neg G), \{\mathbf{T}\}\}$ is adequate. You may use the fact that the sets $\{\neg F, F \wedge G\}$ and $\{\neg F, F \vee G\}$ are adequate.

Write your answer on a blank sheet of paper.

Question 3. (15 points)

Prove or disprove: If $F \vee \neg G$ is satisfiable, then $(F \longrightarrow H) \longrightarrow (G \longrightarrow H)$ is satisfiable. First you must write Proof or Disproof and then provide the proof or the counter-example.

Write your answer on a blank sheet of paper.

Question 4. (14 points)

Construct a derivation tree of $\square$ from $S = \{\{\neg A, \neg B, C\}, \{A, C, E\}, \{\neg C, D\}, \{\neg C, \neg D\}, \{\neg E, F\}, \{\neg E, \neg F\}, \{B, G\}, \{B, \neg G\}\}$.

Draw the derivation tree on a blank sheet of paper.

Question 5. (19 points)

Apply the algorithm given in the book to find a CNF for
$F = \neg\{\neg[(A \vee (B \vee C)] \longleftrightarrow (\neg C \wedge D)\}$.

Show your work on a blank sheet of paper.