COT 3420
Section U2
SPRING 2007

EXAM # 1

INSTRUCTIONS

1. This test is open book, open notebook.
2. There are 5 questions on the test, for a total of 100 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
4. Circle the answers to question 1 on the exam paper. Write the answers to the other questions on blank sheets of paper.
5. If you do not understand the meaning of a question ask me during the test.
6. You have 1 hour to work on the test.
7. If you took the induction test, you get 5 extra points.
8. Write your name below.

NAME: _

QUESTIONS

Question 1.(28 points)

For each of the following 14 statements select the string that not only makes the assertion true, but also states the strongest possible result. There is no penalty for wrong guessing, but choose only one answer.

1. If $\bigvee_{i=2}^{4} F_i = \ldots$
   a. $((F_2 \vee F_3) \vee F_4)$.
   b. $(F_2 \vee (F_3 \vee F_4))$.
   c. $\mathbf{T}$.
   d. $\Box$.

2. If $F \models G$, then ...
   a. $Mod[\{F\}] \subseteq Mod[\{G\}]$.

b. $Mod[\{G\}] \subseteq Mod[\{F\}]$.

c. for some F and G, neither $Mod[\{F\}] \subseteq Mod[\{G\}]$ nor $Mod[\{G\}] \subseteq Mod[\{F\}]$ holds.

3. Let S be a satisfiable set of formulas. If $Mod[S]$ is finite, then ...

a. S must be infinite.

b. every atom must occur in some formula of S.

c. all but a finite number of atoms must occur in S.

d. S must contain an infinite number of atoms.

4. If $\models (F \longleftrightarrow G)$ and F is satisfiable, then ...

a. G is satisfiable.

b. G is unstisfiable.

c. G can be satisfiable or unsatisfiable.

5. $n[con, (\neg((P_0 \vee \neg(P_1 \wedge P_2)) \vee (P_3 \longrightarrow P_4)) \wedge P_5)))] =\ldots$

a. 2.

b. 4.

c. 5.

d. 7.

6. ...is not a prefix of **Shrek**.

a. λ

b. **Shrek**

c. **Shr**

d. **rek**

7. In the CNF form $F = (((P_0 \wedge ((P_1 \vee P_2) \vee P_3)) \wedge P_4) \wedge (P_5 \vee P_6))$, $L_{2,1}$ ...

a. does not exits.

b. P_1.

c. P_2.

8. If the string S is both a prefix and a suffix of formulas, then ...

a. $n[con, S] = n[(, S]$.

b. $n[con, S] \leq n[(, S]$.

c. $n[con, S] \geq n[(, S]$.

d. both $n[con, S] = n[(, S]$ and $n[con, S] \neq n[(, S]$ hold because there are no such strings.

9. The index of an equivalence relation is ...
   a. the size of domain of the equivalence relation.
   b. the number of equivalence classes.
   c. the size of the largest equivalence class.

10. If 3.2 is an address in a tree, then ... must also be an address in the same tree.
   a. 4
   b. 3.3
   c. 2.1
   d. 3.1

11. With n atoms we can define ... non-equivalent formulas.
   a. n
   b. $2n$
   c. 2^n
   d. 2^{2^n}
   e. $3^n + 1$.

12. Let S be a finite set of clauses. $Res^{n+1}[S] - Res^n[S]$ is ...
   a. the set of clauses that have S-derivation trees of height $n + 1$.
   b. the set of clauses that have minimal S-derivation trees of height $n + 1$.
   c. the set of clauses that have S-derivation trees of height $\leq n + 1$.

13. If $S = \{(F \wedge \neg G), \mathbf{T}\}$ be a set of connectives. Then, ... is an S-formula.
   a. $(P_0 \vee P_1)$
   b. $(P_0 \wedge P_1)$
   c. $(\mathbf{T} \wedge \neg P_1)$
   d. $(\neg P_1 \wedge \neg P_2)$

14. Let S and T be two satisfiable sets of formulas. If $S \cup T$ is unsatisfiable, then ...
   a. for some $F \in S$, $\neg F \in T$.
   b. $\square \in Res^*[S] \cup Res^*[T]$.
   c. $\square \in Res^*[S \cup T]$.

Question 2. (25 points)

Prove by structural induction that

every non-empty suffix S of F that satisfies the equation $n[(, S] = n[), S]$ is a formula.

Hint: use the lemma that says that every suffix U of a formula G satisfies the inequality $n[), U] \geq n[(, U]$.

Write your answer on a blank sheet of paper.

Question 3. (15 points)

Proof or disproof: If $F \vee G$ is a tautology, and $\neg G$ is satisfiable, then F is satisfiable.

First you must write if you go for the proof or for disproof. Then, write the proof or display the counterexample on a blank sheet of paper.

Question 4. (14 points)

Construct a derivation tree of $\square$ from

$S = \{\{A, B, C, D\}, \{\neg A, \neg E\}, \{\neg C, F\}, \{\neg A, E\},$

$\{\neg B, D\}, \{\neg D, G\}, \{\neg C, \neg F\}, \{\neg D, \neg G\}\}.$

Draw your tree on a blank sheet of paper.

Question 5. (19 points)

Apply the algorithm given in the book to find a CNF for

$F = \neg[(A \wedge \neg(B \vee C)) \longleftrightarrow \neg(B \vee D)].$

Show your work on a white sheet of paper.