COT 3420
SPRING 2007
Section U1

EXAM # 2

INSTRUCTIONS

1. The exam is open book, open notebook.
2. There are 7 questions on the test, for a total of 100 points.
3. For Question 1, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour and 15 minutes to work on the test.
5. Write the answers to questions 1, 3, 4, 5, and 6 on the exam paper. Write the other answers on the blank sheets.
6. Print your name below.

NAME: _

QUESTIONS

Question 1. (27 points)

The universe of structure $\mathcal{A}$ is $\{3, 4, 5, 6\}$. The $\mathcal{A}$ interpretations of a, x, and y are $a^{\mathcal{A}} = 3$, $x^{\mathcal{A}} = 5$, $y^{\mathcal{A}} = 6$. The tables for the functions $f^{\mathcal{A}}$ and $g^{\mathcal{A}}$ and the predicate $P^{\mathcal{A}}$ are shown in Figure 1. Evaluate the terms and formulas below. Do not show your work, just write the answer after the equal sign.

1. $\mathcal{A}[f(x)] =$

2. $\mathcal{A}[g(x, y)] =$

3. $\mathcal{A}[g(g(a, y), f(x))] =$

4. $\mathcal{A}[P(x, f(a))] =$

u	$f^{\mathcal{A}}[u]$
3	4
4	5
5	6
6	3

$f^{\mathcal{A}}[u]$

$u \backslash v$	3	4	5	6
3	3	4	5	6
4	4	4	4	4
5	3	5	5	6
6	3	5	6	4

$g^{\mathcal{A}}[u, v]$

$u \backslash v$	3	4	5	6
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1
6	1	0	0	0

$P^{\mathcal{A}}[u, v]$

Figure 1: Tables for Question 1

5. $\mathcal{A}[E(f(y), g(x, a))] =$

6. $\mathcal{A}[\forall x P(x, f(x))] =$

7. $\mathcal{A}[\forall y P(f(y), y)] =$

8. $\mathcal{A}[\forall x \exists y P(y, x)] =$

9. $\mathcal{A}[\exists x \forall y \neg P(x, y)] =$

Question 2. (20 points)

Show that the set of connectives $S = \{\forall x(F \wedge \neg G) \wedge \forall y H\}$ is adequate for FOL.

Hint: Let $\sigma[x, F, G, y, H] = \forall x(F \wedge \neg G) \wedge \forall y H$. Find the S meta-formulas $\phi_{\neg}[F]$, $\phi_{\wedge}[F, G]$ and $\phi_{\forall x}[F]$ such that

$\phi_{\neg}[F] \equiv \neg F$, $\phi_{\wedge}[F, G] \equiv (F \wedge G)$ and $\phi_{\forall x}[F] \equiv \forall x F$,

and use the fact that $T = \{\neg F, F \wedge G, \forall x F\}$ is adequate.

Write your answer on a blank sheet of paper.

Question 3 (12 points)

Rectify the formula F.

$F = \forall x[\forall x(P(x, y) \vee Q(z, x)) \wedge \forall z(\neg P(z, x) \vee Q(y, z))] \vee \forall y \exists x[\forall x(\neg P(a, x) \vee \neg Q(y, x)) \wedge \exists y(\neg P(y, x) \vee \neg Q(y, b))]$

Write your answer below.

Question 4 (8 points)

Skolemize the formula

$F = \exists x \forall y \forall z \exists u \forall v \exists w[(P(x, y) \vee P(a, f(z))) \wedge (\neg P(h(v, x), b) \vee \neg P(u, w))]$.

Don't show your work, just write the Skolemized formula.

Write the substitutions as [variable/term]. Display the answer below.

Question 5 (10 points)

Find a prenex form for

$F = [\exists x(\forall y P(x, y) \vee \forall z Q(x, z)) \wedge \forall u(\exists v \neg P(u, v) \vee \exists w \neg Q(w, u))]$.

Don't show your work, just write the prenex form. Display your answer below.

Question 6 (3 points)

Find an existential closure of the formula $F = \exists y \forall v P(x, y, z, u, v)$.

Write your answer below.

Question 7 (20 points)

Let $\forall x \forall y F$ be a rectified formula. Prove that $\forall x \forall y F \models \forall x \forall y F[x/f(x, y), y/g(x, y)]$.
Write your answer on a blank sheet of paper.

Hint: Apply the relabeling lemma and the translation lemma.